Cardinal Sins? Conspicuous Consumption, Cardinal Status and Inequality

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Abstract

This paper analyzes the social dilemma arising when a large population of individuals with differing incomes have concerns over relative deprivation in terms of visible or conspicuous consumption. These relative concerns are cardinal - people care about the size of the gap between own and others' consumption - and include inequity aversion, where negative comparisons are more important than positive, rivalrous preferences, and comparison with mean consumption. The resulting Nash equilibrium is inefficient, with consumption generally exceeding the socially efficient level. In this model, the income distribution has a direct effect on behavior and under rivalrous preferences, an increase in incomes for the rich can raise consumption at all income levels and make almost everyone worse off.

Keywords: relative comparisons, relative deprivation, games, social status, conspicuous consumption.

JEL Classifications: C72, D63, D91.

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1 Introduction

People have relative concerns. They compare themselves with others and in particular compare their consumption levels in terms of highly visible goods such as cars, houses, and clothes. There is considerable evidence both at the individual and at the society level for such social comparisons (Clark et al., 2008), just as there is evidence for social or other-regarding preferences from laboratory studies (Cooper and Kagel, 2016), and empirical evidence on conspicuous consumption (Charles et al. 2009, Heffetz 2011, Jinkins, 2016). In the presence of such externalities, theory predicts substantial inefficiencies, which has the striking implication that taxes could be welfare-improving rather than a deadweight loss. Further, given that people are influenced by what others have or consume, changes in inequality can directly affect behavior and welfare (Hopkins and Kornienko, 2004). Thus, given the recent growth in inequality, understanding these phenomena is important both for welfare analysis and policy design.

However, there is currently a gap between empirical findings and existing theoretical models. From Fehr and Schmidt (1999) onwards, experimental research has provided considerable evidence for asymmetry and cardinality in relative concerns. That is, people care more about negative comparisons with those richer than themselves than comparisons with those who have less. And these concerns are cardinal, people care how much more others have. In contrast, some existing theoretical models, the ordinal model (Frank, 1985; Hopkins and Kornienko, 2004) and the signalling model (Glazer and Konrad, 1996) in effect only look downwards.[1] This is important not only because it implies that existing models omit important psychological aspects of social comparisons, but because, as we will see, it results in completely different predictions. Thus, addressing this gap is vital to explain behavior, understand possible inefficiencies, and to design appropriate corrective policies.

In this paper, I show how to solve large population games between heterogeneous consumers who have cardinal relative concerns. Each must choose how to divide her income between conspicuous and non-conspicuous consumption. Individuals have cardinal preferences over their relative levels of conspicuous consumption, with in particular utility decreasing in relative deprivation in terms of conspicuous expenditure, or equivalently the average expenditure of those above them (envy). I allow for attitudes to those below them to be either negative ("rivalrous" or "competitive" preferences) or positive ("inequity aversion"). A special case is when concerns are negative and symmetric, and then utility simply depends on the average consumption of others (known as "Keeping up with Joneses" or external habit). The model has to be solved simultaneously at all income levels, rather than starting at the lowest income level, which requires new techniques. Nonetheless, it is possible to show, under quite general conditions, that there exists a monotone Nash equilibrium in which conspicuous consumption is strictly increasing in income.

[1] More technically, in auction-like and signalling models, the boundary condition for an equilibrium is given by the behavior of the lowest type - here the poorest agent. But this means that each individual is only affected by the income distribution between that poorest agent and herself.

This Nash equilibrium is not efficient, with consumption mostly higher than in the absence of status concerns. That is, as originally argued in Frank (1985), the pursuit of relative position is a social dilemma. However, the nature of that dilemma differs from the ordinal case. Suppose a planner could choose consumption for each agent to maximize total welfare. I show that the resulting socially optimal consumption allocation is not generally a Pareto improvement on the Nash equilibrium outcome as it is in the ordinal model. Instead the rich are typically worse off than in the non-cooperative equilibrium. In effect, the social planner would make the rich pay for the negative externalities they cause which affect those beneath them. The situation is like that of an industrial plant that pollutes a river, only affecting those who are downstream, and suffers no ill effects itself. Thus, being made to pay for this pollution must make the polluter worse off. However, when there are also negative downward concerns, which generate a social dilemma of excessive consumption at all income levels, there is a possible Pareto improvement as in the ordinal case, but in general this is not utilitarian optimal.

Further, I find that in this setting the distribution of income has a direct effect on behavior and hence on welfare. An increase in relative deprivation, caused by an increase in the incomes of the rich, under rivalrous preferences can lead to increased consumption and lower utility for the middle classes - even though their incomes are unchanged. Further, an increase in inequality under upward looking comparisons can lead to increased consumption by many and a reduction of welfare at all income levels. That is, the effect of greater inequality under cardinal relative concerns is quite different from that under standard assumptions and is directly opposed to that under ordinal relative concerns. This shows that the effect of greater inequality is not obvious and depends heavily on which relative concerns are assumed.

Many others have looked at the question of conspicuous consumption. Frank (1985), Hopkins and Kornienko (2004, 2009) and Becker et al. (2005) analyse the case of ordinal preferences, where individuals care about their rank in the distribution of conspicuous consumption. Turning to cardinal preferences, the most common approach has been to look at the case where individuals care about the difference between their consumption and average consumption in society, a formulation known as "Keeping Up with the Joneses" (KUJ) preferences. But prominent papers in this literature, for example Galí (1994), assume identical agents. Clark and Oswald (1998) and Barnett et al. (2019) look at comparative statics when individuals have KUJ-like preferences. Bilancini and Boncinelli (2012) also contrast ordinal and cardinal status concerns with inequality, but only consider two levels of income. There is a further branch of literature including Ireland (2001), Charles et al. (2009), Heffetz (2011), Moav and Neeman (2012) and Jinkins (2016) that use signalling models. In these signalling models, inequality would normally have a similar effect as in the ordinal status model: signalling/consumption decreases with greater inequality.

The only other papers, to my knowledge, to analyse conspicuous consumption with asymmetric cardinal preferences are Friedman and Ostrov (2008), Bellet and Colson-Sihra (2018) and Bramoullé and Ghiglino (2022). Friedman and Ostrov consider the case where agents are ex ante identical, rather than the heterogenous case considered here.

Bellet and Colson-Sihra (2018) show that conspicuous consumption will be increasing in relative deprivation (similar to Lemma 1 here), and test this result with data from India. Very recently, Bramoullé and Ghiglino (2022) take a similar approach to this paper but in the context of networks. Frank et al. (2014) consider upward-looking relative concerns but directly assume effects on consumption behavior. Thus, this is the first paper that contains analytic results on a game of status with either KUJ or asymmetric preferences under full heterogeneity and thus is able to address the effect of inequality.

Relative deprivation, crucial to this paper, was first formalized by Yitzhaki (1979). Preferences in which individuals care about relative deprivation have been extensively analysed in the context of laboratory experiments (Fehr and Schmidt, 1999, and an enormous subsequent literature). The clear difference is that this literature on social preferences defines utility over money outcomes of an individual and those she compares herself with, while here the preferences are over consumption. The justification for this is simply that consumption is more visible than income and more likely to be the cause of invidious comparisons.

Turning to recent empirical studies, Charles et al. (2009), Heffetz (2011), Jinkins (2016), Bellet and Colson-Sihra (2018) and Lewbel et al. (2022) have very different methodologies but all find evidence for conspicuous consumption being an important phenomenon. Frank et al. (2014), Drechsel-Grau and Schmid (2014), Alvarez-Cuadrado et al. (2016) and Bertrand and Morse (2016) find evidence for relative consumption effects. Chai et al. (2019) more specifically find support for the ordinal status model. Perhaps the only paper that tests directly between ordinal and cardinal preferences is Brown et al. (2008) which finds that a range-frequency model that incorporates both cardinal and ordinal measures is the best fit to their data. However, note that Drechsel-Grau and Schmid's (2014) and Bertrand and Morse's (2016) findings that consumption of the rich affects the consumption of the non-rich are in contradiction to the ordinal model which in effect assumes that people only look downwards.

Finally, both relative concerns and negative externalities can be found in contexts outside conspicuous consumption. Azmat and Iriberri (2010) and Tincani (2018) investigate relative concerns as an incentive for educational performance. Gitmez et al. (2020) note that the negative externalities from risky behavior during the coronavirus pandemic have a similar form to those from conspicuous consumption. Therefore, obtaining a better understanding of negative externalities and how they interact with inequality may be timely.

2 The Model

Standard economic theory treats choice of consumption as a single-agent decision problem. However, if individuals compare their consumption with that of others, the decision becomes strategic because the actions of others affect the outcome of the individual. This strategic approach is taken in Frank (1985), Hopkins and Kornienko (2004) and

Becker et al. (2005), but with a crucial difference. This earlier work assumed ordinal status concerns - satisfaction depends on how an individual ranks in consumption. Here, relative concerns are cardinal, depending on the difference between own consumption and that of others. Further, concerns can be asymmetric with greater weight placed on upwards comparisons.

I consider a large population of individuals who all possess similar relative concerns but who differ in income z. Income is distributed according to the exogenous distribution $G(z)$ on $[\underline{z}, \bar{z}]$, where $\underline{z} > 0$, with continuous non-zero density $g(z)$ and mean μ. As in standard Bayesian games, nature moves first and informs each player of her income level which is her private information, whereas the distribution of income is common knowledge. Then, all simultaneously decide how much income z to spend on visible consumption x, with the remainder $z-x$ spent on other non-visible consumption y.[2] The alternative interpretations, under slightly different assumptions, are that x is consumption and y is leisure or savings (see Section 5.3 below). In any case, let the resulting consumption choices x be aggregated into the distribution $F(x)$, which thus is endogenous.

The next step is to construct a cardinal measure of relative position. Let relative deprivation in consumption of an individual who has visible consumption x facing visible consumption of others x_{-i} be,

$$D(x; x_{-i}) \equiv \int_x^\infty (t - x)\ dF(t) = d(x; x_{-i}) - x(1 - F(x)), \tag{1}$$

where

$$d(x; x_{-i}) = \int_x^\infty t\ dF(t). \tag{2}$$

That is, $d(x; x_{-i})$ is the total expenditure of those having greater consumption than x. Thus the relative deprivation of an individual consuming x is equal to the average distance between x and the consumption levels higher than x. Similarly, define relative advantage as,

$$A(x; x_{-i}) \equiv \int_0^x (x - t)\ dF(t) = xF(x) - a(x; x_{-i}) \tag{3}$$

where

$$a(x; x_{-i}) = \int_0^x t\ dF(t). \tag{4}$$

That is, $a(x; x_{-i})$ is the total expenditure of others who have consumption lower than x. So, relative advantage is equal to the average distance between x and the consumption levels lower than x. This formalization of relative deprivation was introduced, in terms of incomes, by Yitzhaki (1979). The current formulation of relative deprivation and

[2]Heffetz (2011) studies empirically the visibility of different categories of consumption and finds that cigarettes, cars and clothes are the most visible, while insurance and underwear are the least. Bellet and Colson-Sihra (2018) find that in India relative concerns increase demand for goods such as clothing, dairy products, meat, fuel and lighting, packaged products and drinks, and shift demand away from cheap nutritious goods such as cereals, pulses and vegetables.

advantage is inspired by Fehr and Schmidt (1999) (extended to a continuum population in Deaton, 2003).[3]

Let status in consumption for an individual i with visible consumption x facing others' visible consumption x_{-i} be

$$S(x; x_{-i}) \equiv -\alpha D(x; x_{-i}) - \beta A(x; x_{-i}). \tag{5}$$

The idea is that, in assessing her own consumption, the individual places weight α on upward or negative comparisons and weight β on downward or positive comparisons.[4] Note that the derivative of S with respect to own consumption x is $S_x(x; x_{-i}) = \alpha(1 - F(x)) - \beta F(x)$.

Assume that $1 \geq \alpha \geq 0$, $1 \geq \beta \geq -1$, and $\alpha \geq |\beta|$. That is, status is decreasing in relative deprivation but may be decreasing or increasing in relative advantage. Further, upward comparisons are generally stronger than downward comparisons, and relative comparisons are less strong than the weight placed on own consumption.[5] There are four important cases.

1. Inequity aversion: $\alpha \geq \beta > 0$. Status is decreasing in relative deprivation and relative advantage.
2. Neoclassical baseline: $\alpha = \beta = 0$. No relative concerns.
3. Upward comparisons only: $\alpha > \beta = 0$. Status is decreasing in relative disadvantage only.
4. Rivalrous or competitive comparisons: $\alpha > 0 > \beta$. Status decreases in relative deprivation but is increasing in relative advantage.

Where β is positive so that an agent dislikes advantage, or has "compassion" for those lower than her or "guilt", this is inequity aversion as in Fehr and Schmidt (1999). Note that usual additional assumption $\alpha \geq \beta$ implies social loss aversion - negative comparisons are felt more strongly than positive. In contrast, when β is negative, so that an individual has "pride" in being higher up than the poor, then I refer to this as competitive or rivalrous concerns.[6] Here status depends on relative deprivation and

[3]However, Yitzhaki defines a different counterpart to relative deprivation called relative satisfaction, in current notation, $\int_0^x (1-F(t))\, dt = x - A(x, x_{-i})$. This was a natural approach given that he defined D as $\int_x^\infty (1 - F(t))\, dt$ (despite appearances one can check that this is equivalent to (1)). In any case, both Yitzhaki and Fehr and Schmidt assume utility or satisfaction is decreasing in A.

[4]Thus, relative satisfaction might be a better descriptor for S. I use "status" because of its long association with conspicuous consumption.

[5]As we will see below in Proposition 1, for a monotone equilibrium to exist, α and β will typically need to be rather smaller than 1.

[6]Experimental studies (Engelmann and Strobel, 2004; Iriberri and Rey-Biel, 2013) find a range of social or distributional preferences to be present in subject populations. Drechsel-Grau and Schmid (2014) find that, in an empirical study on consumption behavior, comparisons are only upward looking. Experimental research also explores dynamic aspects, such as reciprocity, which in turn can be modelled using psychological game theory (see, for example, Battigalli and Dufwenberg, 2020). Here the model is static and so it abstracts away from such issues.

relative advantage in terms of visible consumption and not income. The justification for this is that, first, visible consumption is literally more visible than income and, second, plausibly negative comparisons follow from seeing the consumption of the rich, not just from them having income.

A special case is when, under rivalrous preferences, one sets $\beta = -\alpha < 0$. Note that

$$A(x; x_{-i}) - D(x; x_{-i}) = \int_0^\infty (x - t)\ dF(t) = x - \int_0^\infty t\ dF(t) = x - \mu_X, \qquad (6)$$

where μ_X is average expenditure on x. Further, $a(x; x_{-i}) + d(x; x_{-i}) = \mu_X$. Thus, when $\beta = -\alpha < 0$, status (5) becomes

$$S = \alpha(x - \mu_X). \qquad (7)$$

This kind of relative concern that depends on the average consumption of others is known as "Keeping Up With the Joneses" (KUJ) preferences (Galí, 1994). The ERC (equity, reciprocity, and competition) model of relative concerns due to Bolton and Ockenfels (2000) is also based on the average of others.

However, individuals have preferences over more than just status. Specifically, take utility to be $U(x, y, S)$, utility is increasing in visible consumption x, non-visible consumption y and status, S. Note that the good x is valued both in terms of its absolute consumption as well its contribution to status S. For example, a car is useful for transport as well as possibly giving prestige. Further, I assume a series of conditions on the utility function, similar to those in Hopkins and Kornienko (2010), that will enable the derivation of a monotone equilibrium and clear welfare results. (i) U is twice continuously differentiable (smoothness); (ii) $U_x(x, y, S) > 0$, $U_y(x, y, S) > 0$, $U_S(x, y, S) > 0$ (monotonicity); (iii) $U_{xy}(x, y, S)$, $U_{ys}(x, y, S) \geq 0$ (complementarity); (iv) $U_{ii}(x, y, S) \leq 0$ for $i = x, y, S$ (own concavity); (v) $U_{xy} - U_{yy} > 0$ (strict normality); (vi) $U_x(x, z - x, S) - U_y(x, z - x, S) = 0$ has a unique solution $x = \gamma(z, S) \in (0, z)$ and it holds that $(U_{xs}(x, z - x, S) - U_{ys}(x, z - x, S))(x - \gamma(z, S)) < 0$. Finally, define the "privately optimal" consumption as $\hat{x}(z) = \gamma(z, 0)$, the level of consumption an individual would chose in the absence of status, for example when $\alpha = \beta = 0$. Condition (v) ensures that γ is strictly increasing in income z, so that good x is strictly normal in the absence of relative concerns. The last condition (vi) seems somewhat complicated but it is automatically satisfied if utility is multiplicatively separable in S.[7] It ensures (see Lemma 1 below) that optimal conspicuous consumption is increasing in the consumption of richer others.

The first result is a characterization of how individuals respond to changes in others' consumption. Without any assumptions on the strategies of others, one can differentiate $U(x, y, S)$ with respect to own consumption x to obtain,

$$U_x(x, z - x, S) - U_y(x, z - x, S) + (\alpha(1 - F(x)) - \beta F(x))U_S(x, z - x, S) = 0. \qquad (8)$$

[7]The log preferences introduced below in Section 2.1 below only satisfy this condition for $x > \gamma(z, S)$. I make clear later the only point where this is an issue and show that the result in question follows directly for the log preferences in any case.

If further $F(x)$ is differentiable (this would be the case, for example, if consumption is strictly increasing in income), then the first order condition implicitly defines a reaction function

$$x(z; x_{-i}) = R(z; F(x); \alpha d(x; x_{-i}) - \beta a(x; x_{-i})). \tag{9}$$

I show that, under rivalrous preferences, others' consumption is a strategic complement, but under inequity aversion, changes in consumption by the rich and the poor generate different reactions.[8] Under inequity aversion, only expenditure by the rich is a complement, while expenditure by the poor is a substitute.[9] In contrast, in the well-known model of Fehr and Schmidt (1999) utility is linear and separable in D and A and hence in S. In this case, in contrast to the above result, changes in others' consumption would not change consumption choices - see Section 2.2 below.

Lemma 1. *Let $F(x)$ be differentiable at own consumption x_i. If $\alpha > 0 > \beta$ (rivalrous preferences), then others' consumption is a* ***strategic complement****, $\partial R/\partial d > 0$ and $\partial R/\partial a > 0$, own consumption x_i is increasing in the consumption of richer others and poorer others. If $\alpha > \beta > 0$ (inequity aversion), then, for an individual whose consumption satisfies $F(x_i) < \alpha/(\alpha+\beta)$, $\partial R/\partial d > 0$, own consumption x_i increases in richer others (****strategic complement****), but $\partial R/\partial a < 0$, own consumption x_i decreases in the consumption of poorer others (****strategic substitute****).*

I now look for an equilibrium in which all individuals use a strictly monotone equilibrium strategy $x(z) : [\underline{z}, \bar{z}] \to [0, \bar{z}]$ so that conspicuous consumption is strictly increasing in income z. When $x(z)$ is strictly increasing, it holds that $G(z) = F(x(z))$. That is, an individual consuming $x(z)$ has the same rank in the distribution of consumption as in the distribution of income. Later, Proposition 1 shows such an equilibrium exists and there is only one such strictly monotone equilibrium. Other, not strictly monotone, equilibria are considered later in Section 5.1. The reasons for initially concentrating on monotone equilibria are simplicity and plausibility. As we will see the other equilibria involve some degree of pooling - people with different incomes choosing the same consumption. This would be in conflict with the standard empirical finding that, on average, consumption is strictly increasing in income.

If visible consumption $x(z)$ is strictly increasing then there is a one-to-one relation between income and consumption. Let $d(z)$ and $a(z)$ be the expenditure of those richer and poorer respectively than the income z,

$$d(z) \equiv \int_z^{\bar{z}} x(t)g(t)\ dt;\ a(z) \equiv \int_{\underline{z}}^{z} x(t)g(t)\ dt. \tag{10}$$

This notation emphasizes that the solution is different from the earlier $d(x_{-i})$ and $a(x_{-i})$ where others' expenditure is arbitrary and not necessarily ordered by income. Similarly, let

$$S(x; z) = x(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z), \tag{11}$$

[8] Inequity aversion also induces technical complications such that it is not possible to sign exactly the competitive responses of high ranked agents.

[9] Clark and Oswald (1998), Barnett et al. (2019) alternatively derive differing strategic responses from differences in concavity or convexity of preferences.

and $S_x(x;z) = \alpha(1 - G(z)) - \beta G(z)$. Thus, in the inequity averse case, S reaches an interior maximum at $z^* = G^{-1}(\alpha/(\alpha+\beta))$. In contrast, when $\beta \leq 0$, S is always increasing (see Figure 1). Utility becomes,

$$U(x, y, S) = U\left(x, z - x, x(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z)\right), \tag{12}$$

using the budget constraint $y = z - x$.

One now can differentiate the utility function (12) to obtain the following first order condition,

$$U_x(x, z-x, S(x;z)) - U_y(x, z-x, S(x;z)) + S_x(x;z)U_S(x, z-x, S(x;z)) = 0. \tag{13}$$

The first term is the intrinsic marginal return to visible consumption x, the second is the marginal return to other consumption y and the third represents an additional marginal return to consumption from relative concerns. When S_x is positive (negative), this additional wedge is positive (negative) and so conspicuous consumption x is greater (smaller) than in the absence of relative concerns.

Next, from (10), one can derive the following system of differential equations and boundary conditions,

$$d'(z) = -x(z)g(z),\ a'(z) = x(z)g(z); d(\bar{z}) = 0,\ a(\underline{z}) = 0. \tag{14}$$

where $x(z)$ solves the first order condition (13). The equations (13) and (14) together form a differential-algebraic system, the solution to which is the equilibrium of the game.

The first main result shows the above defines a strictly monotone equilibrium and derives some of its qualitative properties. In particular, while there is only one monotone equilibrium, it is the unique equilibrium only in the KUJ case ($\beta = -\alpha < 0$). In other cases, there can exist partial pooling weakly monotone equilibria that I detail later in Section 5.1. Importantly, the equilibrium individual consumption depends not only on own income z but also on the distribution of income $G(z)$, both because of its direct presence in the first order condition (13) and because it affects the consumption of others which determines $d(z)$ and $a(z)$. Nonetheless, given an explicit utility function and income distribution $G(z)$, one can solve for closed form solutions to (13) and (14) and hence derive an explicit result for $x(z)$ (see, for example, Section 2.1).

Proposition 1. *For α and β sufficiently small, there exists a unique symmetric strictly monotone equilibrium $x(z)$ that solves (13) and (14). It is the unique equilibrium in the KUJ case ($\beta = -\alpha < 0$). Equilibrium status $S(x(z);z)$ is strictly increasing on $(\underline{z}, \bar{z})$ if $\beta \leq 0$, but for $\alpha \geq \beta > 0$ it is increasing on $(\underline{z}, z^*)$ and decreasing on $(z^*, \bar{z})$, where $z^* = G^{-1}(\alpha/(\alpha+\beta))$. Further, comparing equilibrium consumption $x(z)$ to privately optimal consumption $\hat{x}(z)$, (i) if $\alpha > \beta > 0$ then there is a $\tilde{z} \in [z^*, \bar{z})$ such that $x(z) > \hat{x}(z)$ on $[\underline{z}, \tilde{z})$ with $x(z) < \hat{x}(z)$ on $(\tilde{z}, \bar{z}]$; (ii) if $\alpha > \beta = 0$ then $x(z) > \hat{x}(z)$ on $[\underline{z}, \bar{z})$ with equality at $\bar{z}$; (iii) if $\alpha > 0 > \beta$ then $x(z) > \hat{x}(z)$ on $[\underline{z}, \bar{z}]$.*

The restriction on α and β is necessary to ensure a monotone equilibrium for the following fundamental reason: from (11) one can calculate that $S_{xz} = -(\alpha+\beta)g(z)$

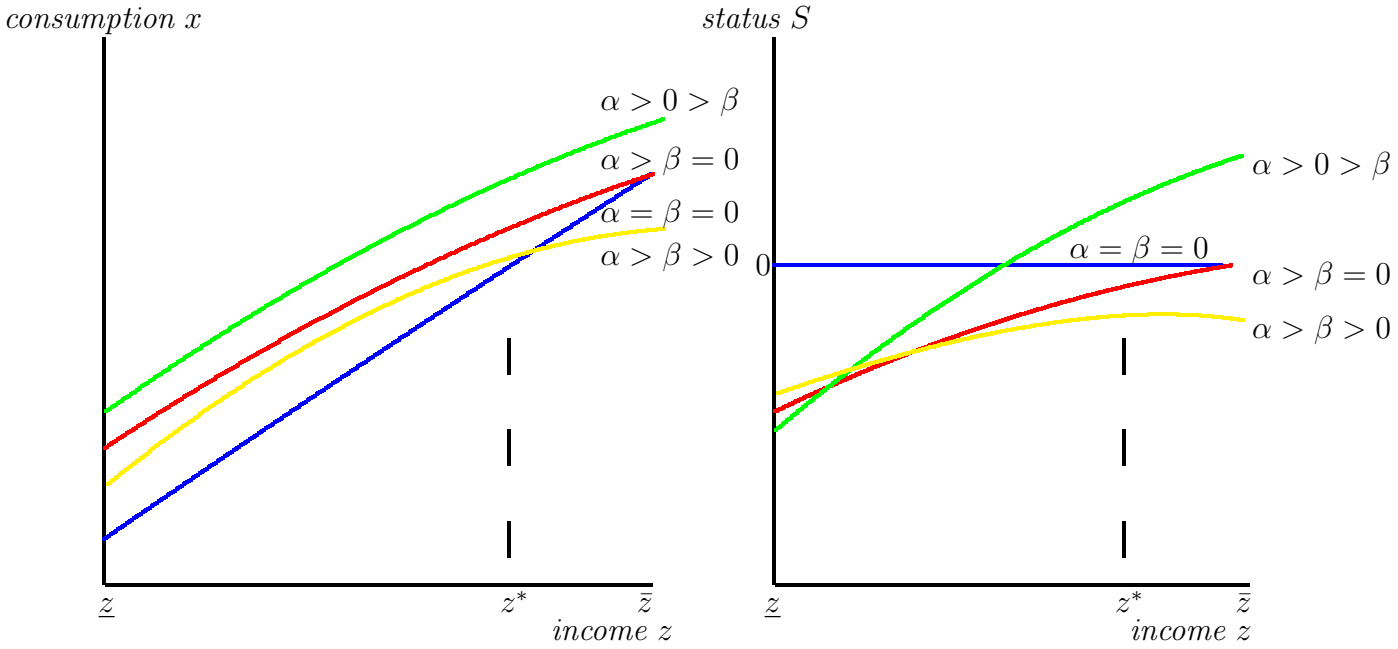

Figure 1: Equilibrium consumption choices (left) and equilibrium status (right) for different parameter values. $\alpha > \beta > 0$ inequity aversion, $\alpha = \beta = 0$ baseline neoclassical or "privately optimal", $\alpha > \beta = 0$ only upward comparisons, $\alpha > 0 > \beta$ rivalrous preferences. The parameter z^*, which equals $G^{-1}(\alpha/(\alpha+\beta))$, is plotted for the inequity aversion case (for the other cases z^* is greater than the maximum income $\bar{z}$).

and thus is strictly negative. Given utility is a function of status S, the standard single crossing condition between action x and type z can fail to hold for α and β sufficiently large. Intuitively, an increase in income could, in the inequity averse case, increase guilt so much that an individual would spend less not more. These issues are not present in the rivalrous case where β is negative so that $\alpha + \beta$ is close or equal to zero.

The qualitative nature of equilibrium is illustrated in Figure 1. Under inequity aversion so that $\alpha > \beta > 0$, equilibrium status $S(x(z); z)$ is increasing for low income levels but achieves a maximum at z^* ($z^* = G^{-1}(\alpha/(\alpha+\beta))$), as guilt becomes the dominant factor. The resulting equilibrium expenditure exceeds the level without relative concerns but at incomes above z^* guilt leads consumption below the privately optimal level. With rivalrous preferences $\alpha > 0 > \beta$, status is always increasing and so equilibrium consumption is always greater than without relative concerns.

2.1 Log Preferences

This section introduces a particular form of log utility that leads to a closed form solution that is useful for some applications. Indeed, this functional form is similar to that used in some of the applied literature on relative consumption effects (Drechsel-Grau and Schmid, 2014; Alvarez-Cuadrado et al., 2016; Bellet and Colson-Sihra, 2018).

Assume

$$U(x, y, S) = \ln[x + S] + \ln[y]. \tag{15}$$

Note that this specific function is strictly monotonic and also satisfies $U_{xx} < 0, U_{yy} < 0, U_{SS} < 0$, $U_{xy} = 0$ and $U_{xS} - U_{yS} < 0$ and therefore fits the earlier general framework

(despite the fact that $U_{xS} < 0$ which is otherwise somewhat unusual in status models). For example, the reaction function can be explicitly derived and is strictly increasing in d. Thus, own consumption is always increasing in richer others' consumption (strategic complements) and increasing (decreasing) in poorer others' consumption if $\beta < 0$ ($\beta > 0$). One has

$$x(z) = R(z; G(z); \alpha d(z) - \beta a(z)) = \frac{z}{2} + \frac{\alpha d(z) - \beta a(z)}{2(1 + \alpha(1 - G(z)) - \beta G(z))}. \quad (16)$$

This reaction function can be combined with the differential equation system (14) to solve for an explicit solution. A special case of this ($\alpha = \beta = 0$) is the privately optimal consumption $\hat{x}(z) = z/2$. Equilibrium utility will be

$$U(z) = \ln[1 + \alpha(1 - G(z)) - \beta G(z)] + 2\ln[\frac{z}{2} + \frac{\beta a(z) - \alpha d(z)}{2(1 + \alpha(1 - G(z)) - \beta G(z))}], \quad (17)$$

where $d(z)$ and $a(z)$ are the solutions to the equation system (14).

The "Keeping Up With the Joneses" (KUJ) case is particularly tractable, with a complete explicit solution obtainable. Combining (7) with the reaction function (16) gives,

$$x(z; \mu_X) = \frac{z}{2} + \frac{\alpha \mu_X}{2(1 + \alpha)}, \quad (18)$$

where μ_X is mean expenditure on x. Integrating this with respect to the income distribution $G(z)$ results in

$$\mu_X = \frac{\mu}{2} + \frac{\alpha \mu_X}{2(1 + \alpha)} \Rightarrow \mu_X = \frac{(\alpha + 1)\mu}{\alpha + 2},$$

where μ is mean income. Thus, the Nash equilibrium strategy is

$$x(z) = \frac{z}{2} + \frac{\alpha \mu}{4 + 2\alpha}, \quad (19)$$

(in this special case, the shape of the income distribution does not matter). It is thus easy to see that consumption is increasing in the average income of others and always higher than in the absence of social preferences. Equilibrium utility is

$$U(z) = \ln[1 + \alpha] + 2\ln\left[\frac{z}{2} - \frac{\alpha \mu}{2(2 + \alpha)}\right]. \quad (20)$$

It is clear that utility is decreasing in average income μ, so that any individual will be worse off if the incomes of others increase.

2.2 Additively Separable Preferences

As noted, in the well-known model of Fehr and Schmidt (1999) utility is linear and separable in D and A and hence in S. In the current notation, one could write,

$$U(x, y, S) = v(x, y) + S(x, x_{-i}) \quad (21)$$

where $v(\cdot)$ is some standard utility function, so that overall utility U is additively separable and linear in status. The first order condition will be similar to before and consumption will be monotone in income under similar conditions to those identified in Proposition 1 - that is, α and β should not be too large.

Note that this specification does not satisfy assumption (vi) and further U is not strictly concave in S. It can be shown that consequently Lemma 1 does not hold. The choice of conspicuous consumption by any individual is **not** affected by the consumption decisions of others. That is, the reaction function (9) will now be a function of income z and rank $F(x)$ alone.

For concreteness, suppose that $v(x, y) = xy$, then one can solve for equilibrium consumption explicitly,

$$x(z) = \frac{z}{2} + \frac{\alpha(1 - G(z)) - \beta G(z)}{2} = \hat{x}(z) + \frac{S_x(x; z)}{2}. \tag{22}$$

That is, consumption is still distorted from the privately optimal level $\hat{x}$ by relative comparisons, in a qualitatively similar way to that illustrated in Figure 1. However, this distortion is now non-strategic and is no longer a function of others' consumption. Thus, the comparative statics results of Section 4 will not apply.

However, as observed by Dufwenberg et al. (2014), equilibria will still not be efficient because of the negative social externalities. One can easily check that the results of Section 3 on socially optimal consumption still apply to the additive case of this subsection.

3 Welfare

The striking result of Frank (1985) is that, with ordinal relative concerns, the Nash equilibrium level of conspicuous consumption is Pareto inefficient. If all individuals simultaneously reduced their consumption in a way that maintained their relative position, everyone would have the same status but higher utility because consumption decisions would be less distorted. The cardinal case is necessarily much more complicated as utility depends on the exact differences in consumption and not just relative position. Thus, while in the ordinal case, the optimal consumption schedule is simply what is privately optimal (the consumption chosen in the absence of relative concerns), here there is no such simple formula and privately optimal consumption $\hat{x}(z)$ is not socially optimal.

Nonetheless, one can show a simple result for rivalrous preferences, where others' consumption is always a negative. In this case, everyone can be made better off if everyone reduces consumption by an equal amount. However, this is not the case under inequity aversion. More generally, I show that under the consumption schedule that maximizes utilitarian welfare either (more likely) the rich or (less likely) the poor are worse off than in Nash equilibrium. The case where $\alpha > \beta = 0$ is particularly clear.

Since people only look upwards, the very rich have a negative externality on others, but suffer little from externalities themselves, so that their consumption choice is not much distorted. A planner would want them to reduce their consumption to reduce the externality on others, but this must make them worse off as they were already at their optimum.

The utility or payoff in a strictly monotone equilibrium will be

$$U(z) = U(x(z), z - x(z), S(z)), \tag{23}$$

where $x(z)$ is the equilibrium function that solves (13) and (14), and $S(z) = x(z)(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z)$, where in turn $d(z)$ and $a(z)$ are the solutions to the equation system (14). Utilitarian welfare is

$$W = \int_{\underline{z}}^{\bar{z}} U(z)\ dG(z). \tag{24}$$

I consider a limited social planner problem in that there is no redistribution.[10] Instead, the question is, keeping the distribution of incomes unchanged, what consumption schedule $x(z)$ would maximize welfare? I show in Lemma 2 below that the relevant first order condition is the following, using the abbreviation $U_x(z)$ for $U_x(x(z), z-x(z), S(z))$ and so on,

$$U_x(z) - U_y(z) + U_S(z)S_x(z) - \alpha k(z) + \beta m(z) = 0, \tag{25}$$

for every $z \in [\underline{z}, \bar{z}]$, where

$$k(z) \equiv \int_{\underline{z}}^{z} U_S(t)g(t)\ dt;\ m(z) \equiv \int_{z}^{\bar{z}} U_S(t)g(t)\ dt. \tag{26}$$

The first order condition (25) is like the Nash first order condition (8) plus two additional terms. The first, $-\alpha k(z)$, reduces consumption to take into account the negative externality of conspicuous consumption by an individual with income z on those with incomes less than z who envy z. The second, $\beta m(z)$, if $\beta > 0$, increases consumption at income z to reduce the guilt felt by those richer than z. If $\beta < 0$, the rivalrous case, then it also will decrease consumption to reduce the negative externality from conspicuous consumption.

Then the utilitarian solution solves the system that combines these differential equations

$$k'(z) = U_S(z)g(z),\ m'(z) = -U_S(z)g(z);\ k(\underline{z}) = 0,\ m(\bar{z}) = 0. \tag{27}$$

with those in (14) and where $x(z)$ now solves the condition (25) rather than (13). To be clear, in general one has to solve simultaneously a system of four differential equations plus the nonlinear first order condition. That is, solving for the utilitarian optimum in the cardinal case is vastly more difficult than in the ordinal case, where it is simply equal to the privately optimal consumption schedule. In any case, denote the solution to the above as $x^*(z)$ and the utility obtained under this allocation as $U^*(z)$.

Lemma 2. *The solution to the system (14), (25) and (27), denoted $x^*(z)$, maximizes utilitarian welfare (24).*

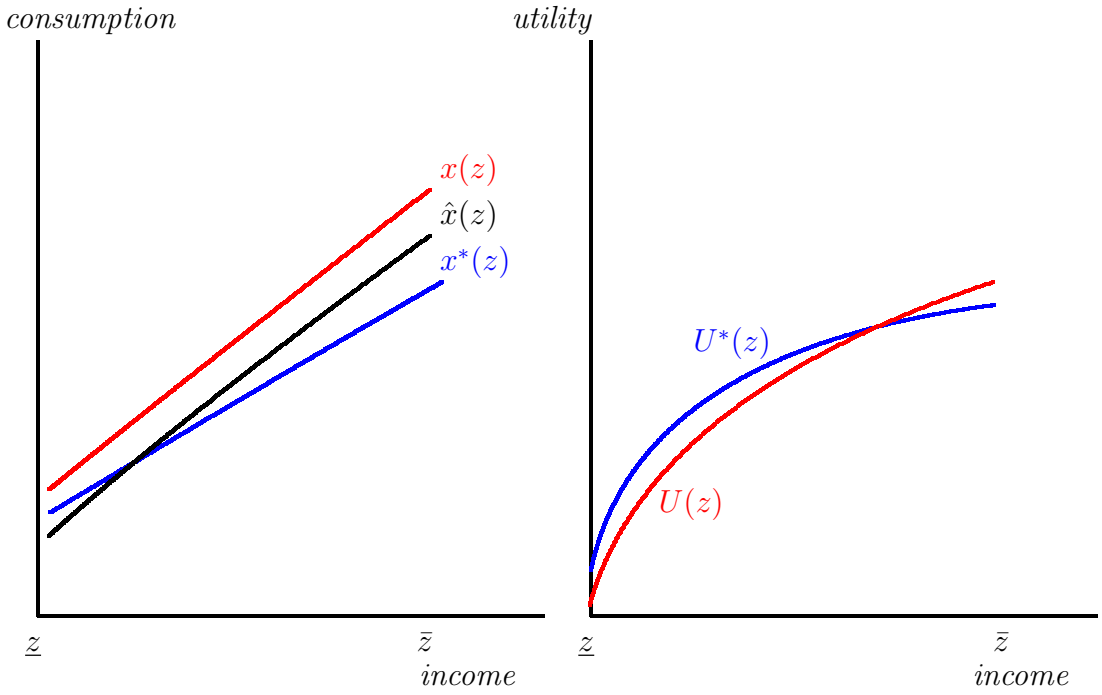

Figure 2: Illustration of Proposition 2: under rivalrous preferences $\alpha > 0 > \beta$, the utilitarian optimal consumption schedule $x^*(z)$ is everywhere lower than the equilibrium schedule $x(z)$ nonetheless it does not equal the privately optimal schedule $\hat{x}(z)$. Utilitarian utility $U^*(z)$ is higher than the Nash equilibrium utility $U(z)$ for most but not for the rich.

Let us start with rivalrous preferences $\alpha > 0 \geq \beta$. The analysis on one level like the ordinal case of Frank (1985) and Hopkins and Kornienko (2004) in that, $x^*(z) < x(z)$, the socially optimal level of consumption is below the Nash equilibrium level at all income levels. This is depicted in Figure 2. However, here the optimal consumption schedule is not simply privately optimal consumption. Rather it is the solution to the system of differential equations derived earlier in this section. Further, imposing the socially optimal consumption does not result in a Pareto improvement. Under upward-looking social comparisons, the rich impose negative externalities on the rest of the population, which the utilitarian solution corrects by heavily pushing down their consumption. This makes the rich worse off.[11]

Proposition 2. *In the rivalrous case ($\alpha > 0 \geq \beta$), the utilitarian optimal level of conspicuous consumption is below the NE level at all income levels so that $x^*(z) < x(z)$ everywhere on $[\underline{z}, \bar{z}]$. For β close enough to zero, the rich are worse off in the utilitarian optimum with $U^*(\bar{z}) < U(\bar{z})$.*

As depicted in Figure 2, the optimal schedule is not equal to the privately optimal. To see this note that the utilitarian solution solves (25) while the privately optimal solves $U_x - U_y = 0$, so the two solutions are equal only when $U_S(z)S_x(z) = \alpha k(z) - \beta m(z)$. This

[10] If redistribution were allowed, one would obtain the well-known result that the utilitarian optimum is complete equality.

[11] One might think that when the weight on upward and downward comparisons are equal such as in the KUJ case, then a Pareto improvement might be possible. I have no general results on this, but I believe this is not the case because even then the lower marginal utility of consumption for the rich will mean that their consumption is severely reduced under the utilitarian optimal scheme.

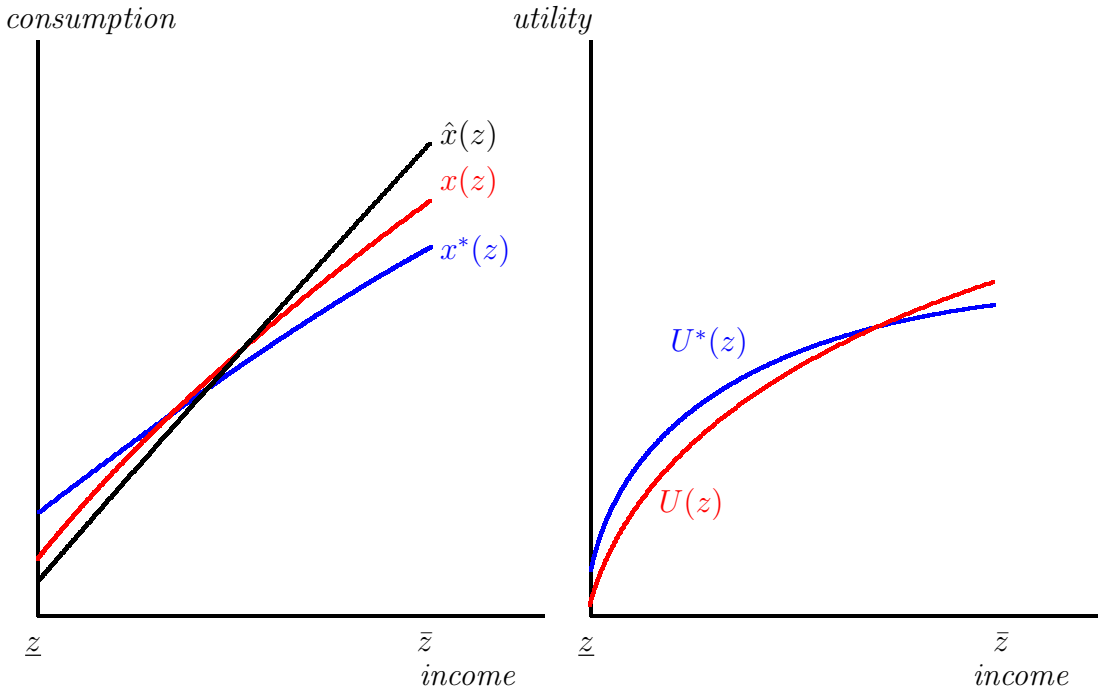

Figure 3: Illustration of Proposition 3: under inequity aversion $\alpha \geq \beta > 0$, the utilitarian optimal consumption schedule $x^*(z)$ is flatter than the equilibrium schedule $x(z)$ and differs further from the privately optimal consumption $\hat{x}(z)$. Optimal utility $U^*(z)$ is higher than the Nash equilibrium utility $U(z)$ for most but not the rich.

will not be true in general. Further, given that S_x is decreasing in z and $\alpha k(z) - \beta m(z)$ is increasing, the utilitarian schedule will be lower than the privately optimal at high incomes.

Moving to the inequity averse case where $\alpha \geq \beta > 0$, it must be the case that optimal consumption $x^*(z)$ is everywhere lower than equilibrium consumption $x(z)$ for small β, because this has just been shown in Proposition 2 for $\beta = 0$. But for larger β, more downward looking compassion, the additional positive term $\beta m(z)$ in (25) can bring optimal consumption above equilibrium consumption at low income levels. Such a case is illustrated in Figure 3. But I show that the optimal consumption schedule is makes the rich worse off than in equilibrium because it pushes their consumption even further below their privately optimal amount. More generally, the socially efficient consumption schedule is not a Pareto improvement on the Nash equilibrium outcome.

Proposition 3. *Under inequity aversion ($\alpha \geq \beta > 0$), one has $x^*(\bar{z}) < x(\bar{z})$ and optimal consumption $x^*(z)$ crosses equilibrium consumption $x(z)$ at most once and from above. Further, either (i) utilitarian average consumption is lower than the equilibrium level $\mu_X^* \leq \mu_X$ and consequently $U^*(\bar{z}) < U(\bar{z})$, the richest individual is worse off under the utilitarian optimal consumption schedule than in the Nash equilibrium; or (ii) $\mu_X^* > \mu_X$ and $U^*(\underline{z}) < U(\underline{z})$, the poorest individual is worse off.*

How could the utilitarian outcome be implemented? The usual suggestion is a Pigouvian tax on conspicuous consumption. This can be integrated into a Mirlees optimum tax framework. See Ireland (2001), Kanbur and Tuomala (2013) for some results in this direction but in signalling and non-strategic settings respectively. A full

analysis of optimal tax in the presence of cardinal relative concerns is left for later research, but note the following. Looking at Proposition 3, in the inequity averse case the effect of the optimal tax would have to be redistributive, raising the consumption of the poor and reducing it for the rich. However, note the important difference from the standard optimal tax framework. There, optimality of redistribution follows from an exogenous social welfare function that places greater weight on the welfare of the poor. Here, reducing inequality in consumption follows purely from concern for efficiency.

Finally, because utilitarian optimal consumption is difficult to calculate, I consider "simple policies". Specifically, what if a planner implemented $x(z) - \epsilon$, so that each individual spent ϵ less on conspicuous consumption x and ϵ more on non-visible consumption y. Then there is a Pareto improvement in the rivalrous case ($\beta < 0$) but not in the inequity averse case ($\beta > 0$). This is simply because this kind of change, where everyone adjusts consumption equally, does not change D or A or, thus, S. But the reduction in x will reduce the distortion towards excess consumption in the rivalrous case and make people better off. However, in the inequity averse case, as shown in Proposition 1, the consumption of the rich is already below the privately optimal level and this simple policy will make them worse off.

Proposition 4. *Compare the equilibrium utility $U(z)$ with the utility $\tilde{U}(z)$ resulting from consumption changing from the equilibrium schedule $x(z)$ to $\tilde{x}(z) = x(z) - \epsilon$ for some $\epsilon > 0$. Then a) if $\alpha > \beta > 0$, then, for ϵ sufficiently small, $\tilde{U}(z) < U(z)$ for $z > \tilde{z}$ for some $\tilde{z} \in [z^*, \bar{z})$; b) if $\alpha > 0 > \beta \geq -\alpha$, then, for ϵ sufficiently small, $\tilde{U}(z) > U(z)$ for all $z \in [\underline{z}, \bar{z}]$.*

However, simple policies that generate Pareto improvements are not in general utilitarian optimal. The utilitarian policy is to reduce the consumption level of the rich more than that of the poor, because of a combination of upward comparisons being more damaging and the lower marginal utility of the rich. This is likely in conflict with the simple policy of a uniform reduction in consumption.

4 The Effects of Greater Relative Deprivation and Greater Inequality

Suppose the distribution of income changes? How does this change behavior and welfare? Here, with relative concerns, equilibrium behavior depends on the incomes of others and so changes to others' resources can have direct effects. For reasons of tractability, in this section, I specialize to the log preferences introduced in Section 2.1.

In general, comparative static results are both more difficult and qualitatively different than under ordinal preferences or in signalling models. For example, Glazer and Konrad (1996) analyse the effects of greater inequality in a signalling model, results which were then applied to conspicuous consumption by Charles et al. (2009). In these signalling models, the equilibrium strategy does not depend on the distribution

of income, but total expenditure can be affected by changes in the distribution due to changes in composition - total expenditure goes up if high spenders increase in relative frequency. The crucial difference in both the ordinal and cardinal games of status is that changes in inequality can have direct effects on behavior - the equilibrium strategy can itself change - as well as there being compositional effects. Further, one can observe, that as in the ordinal status model, signalling equilibria build from the bottom, so changes at the top of the income distribution have no effect on those below. Second, when the marginal propensity to consume is declining (which is the usual assumption), consumption is concave in income, the composition effect necessarily implies that greater inequality will decrease consumption expenditure.[12] Here, I find that both that changes in the income of the rich can affect behavior of the poor and that higher inequality can increase conspicuous consumption.

4.1 Greater Deprivation

In this section, I use the approach of Hopkins and Kornienko (2009) and analyze the effect of greater relative deprivation, comparing at constant rank rather than at constant income. For example, one can compare the choices and outcomes of the median individual before and after a change in the distribution of income. This is useful as it makes clearer who benefits from changes in the income distribution. In particular, I want to investigate what happens when the rich become richer. It is simply not possible to investigate the effect of increased income by making comparisons at a constant income.

First, let us rewrite some of the earlier results in terms of rank. Let $r = G(z)$, an agent's rank in the distribution of income. Let $Z(r) = G^{-1}(r)$ be the inverse distribution of income. Next, if $x(z)$ is the strictly monotone equilibrium strategy in terms of income derived in Section 2, then $x(r) = x(G^{-1}(r))$. Given this monotone relationship between $x(z)$ and $x(r)$, there will exist a monotone equilibrium in terms of rank if and only if there exists one in terms of income. Thus, as pointed out in Hopkins and Kornienko (2009), this is not a new model, it is just a different way of presenting the existing approach.

For what follows, it will be helpful to add that, given $r = G(z)$ one has $g(z)dz = dr$, so that

$$d(r) = \int_r^1 x(t)\ dt;\ a(r) = \int_0^r x(t)\ dt. \tag{28}$$

Status becomes $S(x; r) = x(\alpha(1 - r) - \beta r) - \alpha d(r) + \beta a(r)$. Rewriting the reaction function for log preferences (16) in terms of rank, one obtains,

$$x(r) = \frac{Z(r)}{2} + \frac{\alpha d(r) - \beta a(r)}{2(1 + \alpha(1 - r) - \beta r)}. \tag{29}$$

[12] Heffetz (2011) estimates Engel curves (that is, how consumption changes with income) for a number of goods. There is no particular indication that one should reject (weakly) decreasing marginal propensity to consume even for visible or luxury goods.

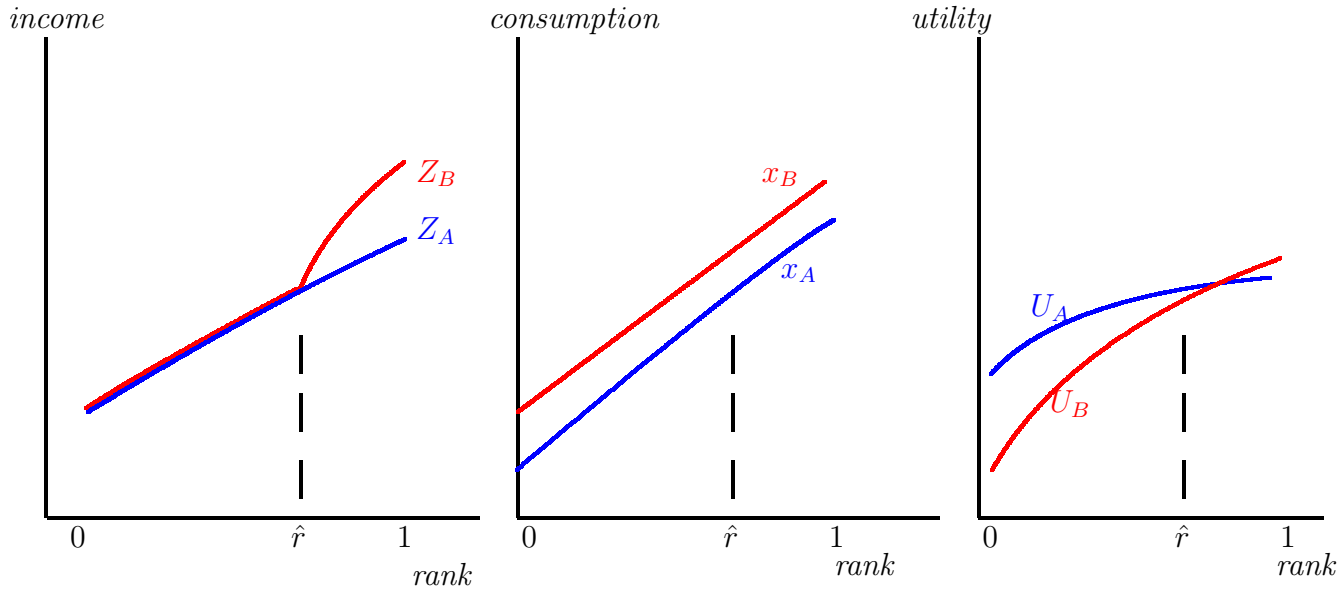

Figure 4: Illustration of Proposition 5: an increase in income only for the rich in inverse income distribution Z_B (first panel), leads to higher conspicuous consumption at all income levels (second panel) and lower utility for most (third panel) evaluated at a constant rank in the income distribution.

Now let us consider a change in the income distribution in which only the rich get richer. Under normal assumptions this of course would represent a Pareto improvement. Here, under the assumption of rvialrous preferences, while the very rich will become better off from the increase in income, even some of the gainers in income will lose in utility. Those whose incomes do not rise are all worse off. The reasons are twofold. First, the increase in income leads the very rich to increase their expenditure, leading to increased relative deprivation for others. Second, this leads everyone to increase conspicuous consumption. It is easy to see this from the reaction function (29) where the increase in consumption by those at the top increases relative deprivation and hence consumption for lower ranked individuals through the $d(r)$ term. The result is illustrated in Figure 4.

Proposition 5. *Suppose $\alpha > 0 \geq \beta \geq -\alpha$ and the rich become richer so that $Z_B(r) > Z_A(r)$ on $(\hat{r}, 1]$ for some $\hat{r} \in (0,1)$ but $Z_A(r) = Z_B(r)$ on $[0, \hat{r}]$, with $Z'_B(r) > Z'_A(r)$ on $(\hat{r}, 1)$. Let $x_A(r)$ and $x_B(r)$ be the resulting monotone equilibrium strategies. Then consumption is higher, $x_B(r) > x_A(r)$, everywhere on $[0,1]$ and utility is lower for all except the very rich, that is $U_A(r) > U_B(r)$ on $[0, \tilde{r}_+)$ for some $\tilde{r}_+ \in (\hat{r}, 1)$ but $U_A(r) < U_B(r)$ on $(\tilde{r}_+, 1]$.*

Note that the above result is impossible under the ordinal concerns model (Hopkins and Kornienko, 2004, 2009) or the signalling models following Ireland (2001). In these other models, the equilibrium strategy at a given income only depends on the distribution of income below that level. Thus changes at the top end of the income distribution will have no effect on utility or behavior of those below.

To expand on this, suppose utility remains to be $U(x, z - x, S)$ but status S is now ordinal so that $S(x, x_{-i}) = F(x)$, one's rank in the distribution of consumption.

Then, the relevant equation that determines equilibrium behavior, following Hopkins and Kornienko (2004, 2009, 2010), in the current notation would be,

$$x'(r) = \frac{U_s(x, Z(r) - x(r), r)}{U_y(x, Z(r) - x(r), r) - U_x(x, Z(r) - x(r), r)}; \; x(0) = \hat{x}(0). \tag{30}$$

This is a differential equation with a boundary condition that the agent with the lowest income and thus holding rank $r = 0$ will spend her privately optimal level of consumption $\hat{x}(0)$. For an individual with higher income, his consumption behavior is determined in effect by integrating the above differential equation from rank 0 to his rank r. Because the differential equation (30) depends on the income distribution through $Z(r) = G^{-1}(r)$, that individual's behavior will be affected by changes in the income distribution - there are relative effects just as in the current ordinal model. However, because the behavior at rank r is determined by solving the differential equation between 0 and r, the behavior at r is only affected by changes in the income distribution on the interval $[0, r]$ and not by any changes in $[r, 1]$. This leads to the following result that, in contrast to the cardinal model, under the ordinal model changes at the top end of the income distribution do not affect behavior below.

Proposition 6. *Suppose the rich become richer so that $Z_B(r) > Z_A(r)$ on $(\hat{r}, 1]$ for some $\hat{r} \in (0, 1)$ but $Z_A(r) = Z_B(r)$ on $[0, \hat{r}]$. Let $x_A(z)$ and $x_B(z)$ be the resulting monotone equilibrium strategies solving the equation for the ordinal model (30). Then consumption is unchanged, $x_B(r) = x_A(r)$, and equilibrium utility is unchanged $U_A(r) = U_B(r)$ for all agents with rank in $[0, \hat{r}]$.*

4.2 Inequality

In this section, I derive some further comparative statics on the effect of greater inequality. I again use the log utility introduced in Section 2.1. Assume income distributions $G_A(z)$ and $G_B(z)$ that have the same support $[\underline{z}, \bar{z}]$. Next one needs a formal definition of being more unequal. As introduced in Hopkins and Kornienko (2004), I use the Unimodal Likelihood Ratio order, defined below, which is a refinement of second order stochastic dominance. That is, if G_A dominates G_B in the ULR order, it is more equal than G_B or stochastically higher than G_B. See for example the first panel of Figure 5.

Definition (ULR): Two distributions (G_A, G_B) satisfy the Unimodal Likelihood Ratio (ULR) order and write $G_A \succ_{ULR} G_B$ if the ratio of their densities $L(z) = g_A(z)/g_B(z)$ is unimodal and $\mu_A \geq \mu_B$. That is, L is strictly increasing for $z < \hat{z}$ and it is strictly decreasing for $z > \hat{z}$ for some $\hat{z} \in (\underline{z}, \bar{z}]$.

The main result of this section finds that greater inequality lowers equilibrium utility under rivalrous preferences. I make this comparison at constant levels of income so that these changes are entirely driven by changes in the negative externalities driven by others' consumption. In particular, the increase of conspicuous consumption at the top end of society has a negative effect on others through an increase in their relative deprivation, and so equilibrium utility falls at most and possibly all income levels. The

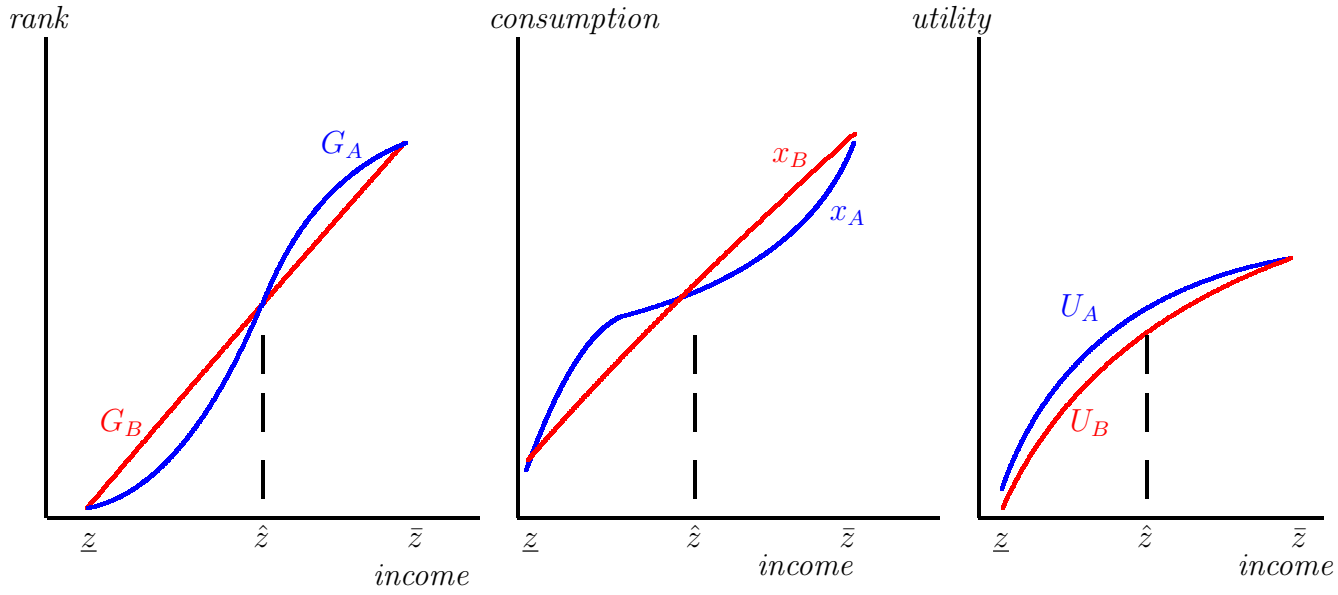

Figure 5: Illustration of Proposition 7: Greater inequality under income distribution G_B (first panel), leads to higher conspicuous consumption for the rich and possibly all (second panel) and lower utility at almost all income levels (third panel).

effect on conspicuous consumption is less clear. While consumption rises for the rich, the poor do not necessarily follow. See Figure 5 for an illustration. The welfare results are therefore not driven by wasteful emulation, rather by the direct psychological effect of higher relative deprivation.

Proposition 7. *Suppose $\alpha > \beta = 0$ and society B is more unequal than A, $G_A \succ_{ULR} G_B$, but A and B have the same mean income $\mu_A = \mu_B$. Let $x_A(z)$ and $x_B(z)$ be the resulting monotone equilibrium strategies. Further, let $\hat{z}_-$ and $\hat{z}_+$ satisfy $\underline{z} \leq \hat{z}_- < \hat{z} < \hat{z}_+ < \bar{z}$.*
(a) In the more unequal society B the rich spend more on consumption than in the more equal society A, but the effect on the poor is ambiguous. That is, first $x_A(z) < x_B(z)$ on $(\hat{z}_+, \bar{z})$; second there can be a further crossing at $\tilde{z}$ in $(\hat{z}_-, \hat{z}_+)$ and if so also a final crossing in $(\underline{z}, \hat{z}_-)$.
(b) Utility is lower for most income levels in the more unequal society, so that for $\tilde{z} < \hat{z}$, it holds that $U_A(z) > U_B(z)$ on $[\tilde{z}, \bar{z})$ and possibly for all z in $[\underline{z}, \bar{z})$.

To give some further intuition for these results, from the first order condition (13), the marginal return to conspicuous consumption depends on S_x which in turn depends on the income distribution. In particular, $S_{xz} = -(\alpha + \beta)g(z)$ so that the marginal return to conspicuous consumption will increase with greater inequality which lowers the density $g(z)$. Thus, greater inequality will tend to raise consumption, which then has spillover effects on others through relative deprivation. However, greater inequality also raises the density at low income levels, which is why consumption does not rise everywhere.

These comparative static results on the effect of greater inequality are almost the complete opposite of those obtained in Hopkins and Kornienko (2004). There, greater inequality reduced competition and increased utility. It is possible to generate similar

results here, but under different assumptions on the form of cardinal preferences. In particular, if $\alpha = 0$ but $\beta < 0$, then individuals only make downward negative comparisons. Perhaps strangely, then the comparative static results are now almost identical to those obtained under ordinal status concerns.

Proposition 8. *Suppose $\alpha = 0 > \beta$ and society B is more unequal than A, $G_A \succ_{ULR} G_B$, but A and B have the same mean income $\mu_A = \mu_B$. Assume $\underline{z} \leq \hat{z}_- < \hat{z} < \hat{z}_+ < \bar{z}$.*
(a) In the more unequal society B the poor spend more on consumption than in the more equal society A. That is, $x_A(z) < x_B(z)$ on $(\underline{z}, \hat{z}_-)$.
(b) In the more unequal society B the poor have higher utility, $U_A(z) < U_B(z) \in (\underline{z}, \hat{z}_-)$.

Let us turn to the KUJ formulation as given in (19). There inequality has no effect on behavior because in general the distribution of income has no effect, only its mean. I give no proof for the result below as it follows from simple observation of (19) and (20).

Proposition 9. *Suppose $-\alpha = \beta < 0$ (KUJ case) and society B is more unequal than A, $G_A \succ_{ULR} G_B$, but A and B have the same mean income $\mu_A = \mu_B$. Then, there is no difference between A and B in consumption, $x_A(z) = x_B(z)$, or utility, $U_A(z) = U_B(z)$.*

In contrast, suppose rather than a change in inequality, one considers an example of unequal growth in which the incomes of the rich increase at greater rate than the rest of society. Then conspicuous consumption rises and utility falls. Again, given that average income rises, the result follows from inspection of (19) and (20).

Proposition 10. *Suppose $-\alpha = \beta < 0$ (KUJ case) and the rich become richer so that $G_B(z) < G_A(z)$ on $(\hat{z}, \bar{z})$ for some $\hat{z} \in (\underline{z}, \bar{z})$ but $G_A(z) = G_B(z)$ on $[\underline{z}, \hat{z}]$, then average income is higher $\mu_B > \mu_A$ which implies that consumption is higher $x_B(z) > x_A(z)$ and utility is lower $U_B(z) < U_A(z)$ everywhere on $[\underline{z}, \bar{z}]$.*

These distributional comparative statics need to be interpreted with care. For example, this final result and Proposition 7 earlier do not in general imply that society A Pareto dominates society B or that its distribution of utility stochastically dominates that in B. The issue is that by changing the distribution some individuals have been made richer and therefore may be better off, even if utility falls at a constant level of income. Simply put, those who gain in income can be better off, but those who see no income gains are made worse off.[13] One can see these issues more clearly by comparison at a constant rank rather at a constant income, which is why I presented the rank-based result, Proposition 5, first.

[13] A simple analysis of equilibrium utility (20) in the KUJ case finds that one is better off if one's own increase in income is greater than the increase in average income multiplied by $\alpha/(1+2\alpha) < 1$.

5 Further Issues and Extensions

5.1 Partial Pooling Equilibria

Up to now, the focus has been on strictly monotone equilibrium. However, in general there are other equilibria. These all involve some degree of pooling in which individuals with different incomes choose the same consumption but this may be combined with a strictly monotone consumption at other income levels. The intuition behind this is that under inequity aversion, there is a tendency to avoid consumption differences and choose similar levels of consumption. But working against this, individuals have different levels of income and, absent status concerns, would choose different consumption levels. Thus, pooling must be over small income intervals. Further, one can show that if β is not too large, then all equilibria will be weakly monotone (so in the rivalrous case ($\beta < 0$), equilibria are necessarily weakly monotone), so all equilibria are broadly qualitatively similar.

More technically, pooling can be incentive compatible because it creates mass points in the distribution $F(x)$ of consumption (if a mass of agents choose the same consumption level $\tilde{x}$, then $F(x)$ is discontinuous at $\tilde{x}$, with $F_{-}(\tilde{x}) = \lim_{x \uparrow \tilde{x}} F(x) < F(\tilde{x})$). While payoffs are continuous in x at such mass points, the status function is not differentiable there, with the right derivative lower than the left (except in the KUJ case). Thus marginal utility with respect to consumption can be strictly positive to the left and strictly negative to the right of the pooling level, so that there is no incentive to deviate either up or downwards.[14]

The following result characterizes equilibria that are not strictly monotone. They are qualitatively similar to strictly monotone equilibria in that the consumption function $x(z)$ is always continuous and at least weakly increasing. Further, it is similar in its relation to the privately optimal level of consumption. For example, the below result implies that in the rivalrous case ($\beta \leq 0$), equilibrium consumption always exceeds the privately optimal level, because in this case $z^* = G^{-1}(\alpha/(\alpha + \beta))$ is greater than or equal to the maximum income $\bar{z}$. See also Figure 6 below.

Proposition 11. *For β sufficiently small, all equilibrium functions $x(z)$ are weakly monotone and continuous. If $z \leq z^*$, then $x(z) \geq \hat{x}(z)$, equilibrium consumption is above the privately optimal level.*

The amount of pooling supportable in such equilibria is increasing in α and in β. Simply put, a high α deters the lowest income types in a pooling group from deviating downwards because they then would suffer deprivation relative to the group of other agents. Similarly, a high β deters upward deviations from high income types because they would suffer from the advantage relative to the group

More generally, a larger proportion of agents can pool where the income distribu-

[14]This observation that asymmetric relative concerns can induce non-differentiability and hence multiple equilibria was first made by Bhaskar (1990).

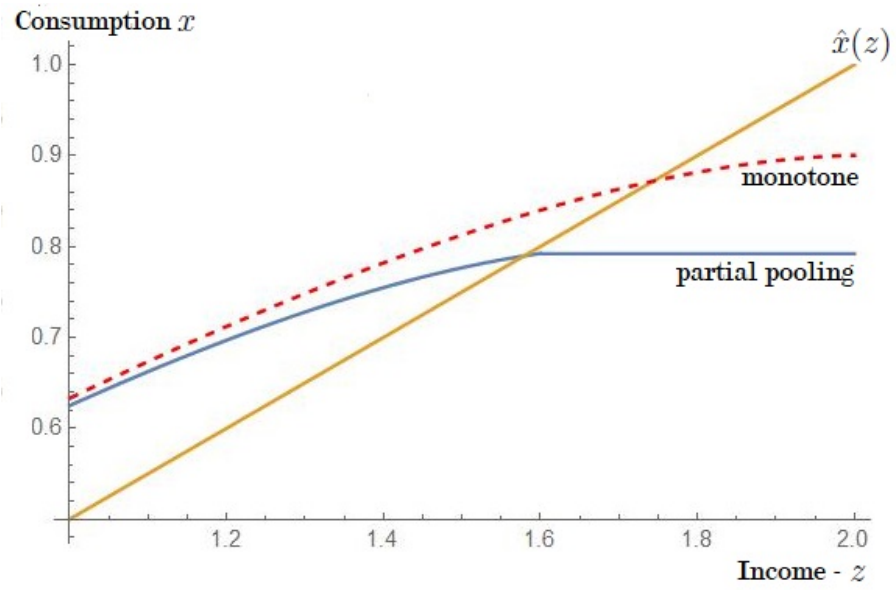

Figure 6: Strictly monotone (dashed; $\alpha = 0.5, \beta = 0.2$) and partial pooling (solid; $\alpha = 0.5, \beta = 0.4$) equilibria plotted against privately optimal consumption $\hat{x}(z)$.

tion is relatively compact so that there are not large differences in absolute income amongst the pooling types. But this shows the limitation of such equilibria. In real populations which have significant income differences between rich and poor, the values of α and β would have to be implausibly large to compel all to choose the same level of consumption. And if instead pooling is limited and local and the equilibrium has strictly monotone components, then such equilibria are qualitatively similar to a strictly monotone equilibrium.

The existence of partial pooling equilibria also partially answers the question about what form do equilibria take when α and β are too large for a strictly monotone equilibrium to exist. Instead, a weakly monotone equilibrium is possible, with pooling at high incomes.

For example, using the logarithmic preferences of Section 2.1, it is possible to construct some numerical examples. Suppose incomes are uniformly distributed on [1,2]. Then, first, assume $\alpha = 0.5$ and $\beta = 0.2$. There is a strictly monotone equilibrium, but one can see in Figure 6 that at high incomes the equilibrium consumption function $x(z)$ is almost flat as the rich, facing guilt as the parameter β is strictly positive, moderate their consumption.

Second, assume now that $\alpha = 0.5$ and $\beta = 0.4$. For these values of the social preference parameters guilt is too strong for there to be a strictly monotone equilibrium. However, there is still a partial pooling equilibrium with pooling at the top - in which all agents with incomes on [1.6, 2] choose $x = 0.792$, while consumption is strictly monotone on [1, 1.6). This is also illustrated in Figure 6. The privately optimal consumption function $\hat{x}(z)$ is also illustrated for comparison. The strictly monotone and partial pooling equilibria are qualitatively similar.

5.2 Heterogeneity in Relative Concerns

One consistent finding of the experimental literature that attempts to measure social preferences is that there is diversity across subjects. For example, Iriberri and Rey-Biel (2013) find that the majority of subjects have some form of social preferences, self-interest (without apparent social preferences) is the biggest single group, with inequity aversion second. Thus, an important question (which has not previously been addressed in the status literature) is what happens when there are heterogeneous preferences?

Since the experimental literature finds that the plurality of subjects have no social preferences, in this section I analyze outcomes where there is a mix of status-conscious and status-neutral individuals. Let θ of the population have status concerns and $1-\theta$ have no status concerns so that they have neoclassical preferences (equivalently for them $\alpha = \beta = 0$). Let both types have the same income distribution $G(z)$. Let the status-concerned individuals use strategy $x(z;\theta)$ and the status-neutral individuals use strategy $\hat{x}(z)$, which as introduced earlier represents optimal consumption in the absence of status concerns. That is, for the status-concerned their own frequency θ will affect their consumption choice through relative consumption externalities. However, for the status-neutral choosing consumption is a single-person decision problem and so neither the population composition nor the consumption of others affects their choice.

If both $x(z;\theta)$ and $\hat{x}(z)$ are strictly increasing, a status-concerned individual consuming x will have rank,

$$F(x;\theta) = \theta G(x^{-1}(x)) + (1-\theta)G(\hat{x}^{-1}(x)) = \theta G(z) + (1-\theta)G(\hat{x}^{-1}(x)). \tag{31}$$

Status is defined as before but with $F(x)$ in (5) replaced with $F(x;\theta)$. Further, one now has,

$$S_x(x;z) = \alpha(\theta(1-G(z)) + (1-\theta)(1-G(\hat{x}^{-1}(x)))) - \beta(\theta G(z) + (1-\theta)G(\hat{x}^{-1}(x))). \tag{32}$$

This implies that S_x is generally smaller as $G(\hat{x}^{-1}(x)) > G(z)$ because $x(z) > \hat{x}(z)$. Thus, conspicuous consumption is increasing in the proportion of the status-conscious θ.[15]

One can then derive the first order conditions for the status-conscious strategy in the same way as before, but replacing the expressions in (13) for S and S_x with the new versions above. The first order condition implicitly defines $R(z; G(z;\theta); \alpha d(z;\theta) - \beta a(z;\theta))$, where

$$d(z;\theta) = \theta \int_{\bar{z}}^{z} x(t)g(t)\,dt + (1-\theta)\int_{\bar{z}}^{z} \hat{x}(t)g(t)\,dt. \tag{33}$$

The derivation of $a(z;\theta)$ is similar. One has

$$d'(z;\theta) = -\theta x(z)g(z) - (1-\theta)\hat{x}(z)g(z);\ a'(z;\theta) = -d'(z;\theta);\ d(\bar{z};\theta) = 0,\ a(\underline{z};\theta) = 0. \tag{34}$$

[15] This also suggests that this is another example where cardinal preferences are very different from ordinal. In the ordinal case, those who are concerned with status should have a greater incentive to consume when they are in a minority as it is easier to rise in rank when others, the status-neutral, spend less.

The main points are that, even under heterogeneous preferences, it is possible to calculate the equilibrium and that it is qualitatively similar to the earlier result when all are status-conscious. Here, an equilibrium will be a pair $(x(z;\theta), \hat{x}(z))$, where $x(z;\theta)$ solves the system (13), (32) and (34), and so is the consumption schedule for the status-conscious individuals. In contrast, $\hat{x}(z)$ solves $U_x(x, z-x, 0) - U_y(x, z-x, 0) = 0$ and is the schedule for the status-neutral. One can also show that $x(z;\theta)$ increases with θ, the proportion of status-conscious individuals.

Proposition 12. *For α and β sufficiently small, there exists a monotone equilibrium $(x(z;\theta), \hat{x}(z))$ in the heterogenous case. In the rivalrous case $\alpha > 0 \geq \beta$, $x(z;\theta)$ is increasing in θ, and $x(z;0) > \hat{x}(z)$ everywhere on $[\underline{z}, \bar{z})$.*

That is, the consumption by the status-conscious in the heterogeneous case is qualitatively similar to the case where all are status-conscious. However, clearly aggregate or average consumption is increasing in the proportion θ of status-conscious. But even when the proportion of status-concerned individuals is small, the status-conscious outspend the status-indifferent . Note that as $\theta \downarrow 0$, $x(z;\theta)$ approaches $R(z; G(z,0), \alpha\hat{d} - \beta\hat{a})$, where $\hat{d}(z) = \int_z^{\bar{z}} \hat{x}(t)g(t)dt$, and does not approach $\hat{x}(z)$. For example, for the log preferences with $\beta = 0$,

$$x(z,0) = \frac{z}{2} + \frac{\alpha\hat{d}(z)}{2(1+\alpha(1-G(z)))}, \tag{35}$$

where $\hat{d} = \int_z^{\bar{z}} \frac{t}{2} g(t)dt = d(z;0)$. So, status-conscious individuals will still spend more on consumption than is privately optimal, even when a vanishingly small proportion of the population.

5.3 Labor Supply

In this subsection, I present an alternative to the main model, which assumed each agent's income was exogenous. Here instead agents choose their labor supply and hence income is endogenous. Nonetheless, it is possible to show that the rat-race results of the main model are preserved. In particular, under rivalrous preferences, labor supply is too high, and it can be driven higher by an increase in wages of the rich.

Let each individual's type now be here productivity w with productivities continuously distributed according to $G(w)$ on $[\underline{w}, \bar{w}]$ with continuous density $g(w)$ and mean μ_w. Assume all can work in a competitive labor market and each has an hourly wage w that is equal to one's productivity. The place of income z in the main model will now be taken by $z = m + wT$ where m is (common) unearned income, w is the wage rate and T is total available hours. Let H be hours of leisure, with the counterpart being that labor supply is $L = T - H$.

Individuals compare themselves on the basis of the variable x, but this now represents total consumption. The place of non-conspicuous consumption is now taken by

leisure H. For simplicity, let us adopt the log utility specification introduced in Section 2.1, so that utility is

$$U = \ln[x - \alpha D(x, x_{-i}) - \beta A(x, x_{-i})] + \ln[H] \tag{36}$$

where D and A are, as before, relative deprivation and relative advantage respectively, constructed by comparing own consumption x with that of others x_{-i}. The budget constraint becomes,

$$w(T - H) + m = px;\ H = \frac{m + wT - px}{w} = \frac{1}{w}(z - px),$$

where p is the price of x. So

$$U = \ln[x - \alpha D(x, x_{-i}) - \beta A(x, x_{-i})] + \ln[z - px] - \ln[w].$$

Given a monotone equilibrium, $x(w)$, we have

$$d(w) \equiv \int_w^{\bar{w}} x(t)g(t)\ dt;\ a(w) \equiv \int_{\underline{w}}^{w} x(t)g(t)\ dt. \tag{37}$$

The solution for x changes from (16) to become,

$$x(w) = \frac{m + wT}{2p} + \frac{\alpha d(w) - \beta a(w)}{2(1 + \alpha(1 - G(w)) - \beta G(w))}. \tag{38}$$

Equilibrium leisure will be,

$$H(w) = \frac{1}{w}\left(\frac{m + wT}{2} - p\frac{\alpha d(w) - \beta a(w)}{2(1 + \alpha(1 - G(w)) - \beta G(w))}\right), \tag{39}$$

and, labor supply,

$$L(w) = \frac{T}{2} - \frac{m}{2w} + \frac{p}{w}\frac{\alpha d(w) - \beta a(w)}{2(1 + \alpha(1 - G(w)) - \beta G(w))} \tag{40}$$

So, labor supply is now affected by relative concerns. Specifically, an increase in α or d will increase labor supply. So any factor that would increase x in the baseline model will tend to increase labor supply in this model. So the rat race result holds both for consumption and work hours.

Proposition 13. *For α and β sufficiently small, there exists a unique symmetric strictly monotone equilibrium $x(w)$. In the rivalrous case $\alpha > 0 > \beta \geq -\alpha$, consumption and labor supply are higher and leisure is lower than both the privately optimal and socially optimal amounts.*

One could also apply Proposition 5 to show that an increase in productivies/wage rates at the top would lead here to increased labor supply as well as increased consumption at all income levels. The idea that individuals respond to richer others by increasing their own labor supply finds empirical support in Luttmer (2005).

5.4 Microfoundations

Why would people have status preferences? Under what circumstances would they be ordinal and when cardinal? While there are several potential reasons for status or relative concerns (see the discussion for example in Hopkins (2008)), the one most commonly discussed in the economics literature is rivalry. In particular, as Cole et al. (1992) showed, if agents are competing in a matching tournament where ordinal relative performance determines marriage matching outcomes, then reduced form preferences will depend on ordinal position.

As already demonstrated by Weibull and Salomonsson (2006) and Bilancini and Boncinelli (2014), when rivalrous competition is subject to noise so that outcomes are stochastic, then the resulting reduced form utility involves *cardinal* relative concerns, not ordinal. To explain the difference, in a purely deterministic tournament, player A will beat player B if her performance is higher by any distance. But in a stochastic tournament, the probability that she wins is increasing in distance by which her performance is higher. So, her expected utility depends on the cardinal difference between her and her rivals.

However, in much work on social preferences, preferences are not just cardinal, they are asymmetric, with differing effect upward and downward comparisons. What would explain this? Weibull and Salomonsson (2006) allow for both rivalry and group selection, individuals both compete within a group and face a joint risk as a group. They show that this implies that an individual's utility could be decreasing in another's resources when the other individual is richer because the rivalry motivation dominates. However, when the other is poorer and faces a survival risk, the group selection motive can be more important, as another failing to survive would weaken the group. Bilancini and Boncinelli (2014) provide a different explanation based on differing types of matching equilibria.

6 Conclusions

This is the first detailed analysis of the interaction between conspicuous consumption and inequality under the assumption of cardinal preferences. We find that the results here depend heavily on which exact form of cardinal preferences are assumed, inequity averse or rivalrous. But in either case, the results are different from those obtained under ordinal preferences by Frank (1985), Hopkins and Kornienko (2004, 2009) or using signalling models such as those in Ireland (2001) or Charles et al. (2009).

One of the main findings is that here there is a negative effect from increased inequality with the possibility of utility falling at every income level. This partially works through increased conspicuous consumption by the rich leading to an increase in consumption by those with lower income. However, the negative effect also works through the consequent increase in relative deprivation, people are worse off because of increased negative relative comparisons.

I hope these results, by contrasting the results under cardinal and ordinal preferences, will be useful in empirical investigations of relative concerns in actual consumption behavior. However, there is one further point about empirical identification of such effects that is not considered sufficiently frequently. Samuelson (2002) points out that relative consumption effects can also be produced by social learning. Seeing others consume could induce consumption through a learning or demonstration effect. However, the big difference in predictions between Samuelson's (2002) learning model and the ordinal status model of Hopkins and Kornienko (2004) was on the effect of inequality. It is welfare-decreasing in the former, but tends to reduce wasteful consumption in the latter. However, given the results here on cardinal status preferences, it is less clear how to test between status and social learning. Equally, if social learning and cardinal status concerns are qualitatively similar in their effects, it may be more important to separate the cardinal and ordinal models.

Finally, welfare analysis is more complicated than in the ordinal case because under cardinal preferences welfare depends on the exact level of others' consumption. It is also more complex in its implications for policy, because of the increased difficulty in implementing Pareto improvements. In general, the rich are worse off with utilitarian optimal consumption than in the non-cooperative equilibrium. With rivalrous preferences, it is possible to construct a Pareto-improving policy, but it will not in general be utilitarian optimal. Further, the effect of inequality is also complex. For example, rank-based comparisons show that the rich gain from greater inequality even if others are worse off. These results may also interact with further behavioural issues. For example, Dal Bó et al. (2017) conduct a laboratory experiment which has a social dilemma similar to that considered here. Similarly, there exists a policy that reduces the externality and hence results in a Pareto improvement, but many subjects vote against it. Thus, policy design and political economy in situations where negative externalities are important, including, as analyzed in Gitmez et al. (2020), the recent coronavirus pandemic, remain a complex and important topic.

Appendix: Proofs

Proof of Lemma 1: This follows from the application of the implicit function theorem. Write the left-hand side of (8) as $\psi(x, z, S)$. Then $\partial R/\partial d = -(\partial\psi/\partial d)/(\partial\psi/\partial x)$. One has $\partial\psi/\partial d = \alpha(U_{yS} - U_{xS} - U_{SS}S_x)$. Now, given $S_x = \alpha - (\alpha+\beta)F(x)$, one has $S_x > 0$ everywhere when $\beta < 0$ and, for $\beta > 0$ for $F(x) < \alpha/(\alpha+\beta)$. When $S_x > 0$, by inspection of (8), one can see that $x > \gamma(z, S)$ and thus by condition (vi) on the utility function, $U_{xS} - U_{yS} < 0$. One also has $\alpha > 0$ and $U_{SS} \leq 0$ by assumption and thus $\partial\psi/\partial d > 0$. Similarly, it is easy to verify that, first if $F(x)$ is differentiable then $\partial\psi/\partial x$ exists, and, second, that $\partial\psi/\partial x < 0$ given the assumptions on $U(\cdot)$. Thus, $\partial R/\partial d > 0$ and the result follows. The derivation of $\partial x/\partial a$ is similar for $z < z^*$. But when $z > z^*$, one has $S_x < 0$ and the general case is ambiguous. □

Proof of Proposition 1: Suppose that

$$\alpha\frac{g(\bar{z})U_S}{U_{yS}+g(\bar{z})U_S}+\beta<\frac{U_{xy}-U_{yy}}{U_{yS}+g(\bar{z})U_S}>0\Rightarrow\beta<\frac{U_{xy}-U_{yy}}{U_{yS}+g(\bar{z})U_S}. \tag{41}$$

for the individual with the highest income $\bar{z}$ and highest rank $G(z)=1$ so that $d(z)=0$ and $a(z)=\mu_X$. Thus, the arguments of U_{xy} etc. should be evaluated at $(x,\bar{z}-x,-\beta(x-\mu_X))$, where μ_X is the average expenditure on x across the population.[16] I will show that this inequality is sufficient for monotonicity.[17]

First, I show that an equilibrium $x(z)$ is necessarily weakly increasing because best responses are increasing in income z. Differentiating the first order condition (8) with respect to income z one obtains,

$$\frac{\partial^2U(x,z-x,S)}{\partial x\partial z}=U_{xy}-U_{yy}+U_{yS}(\alpha(1-F(x))-\beta F(x)). \tag{42}$$

This is greater than zero for $F(x)=0$ and is still greater than zero at $F(x)=1$ for $\beta<(U_{xy}-U_{yy})/U_{ys}>0$ which holds if (41) holds - that is, if β is sufficiently small. This implies that the best response for each agent is (weakly) increasing in own income z.

By definition, in a strictly monotone equilibrium $F(x)$ is continuous and strictly increasing which implies that its inverse $x(F)$ is continuous and strictly increasing. This in turn implies that, given $G(z)$ is continuous and strictly increasing, $x(z)$ is continuous and strictly increasing and that, in equilibrium, $F(x)=G(z)$. Thus, (13) is continuous and differentiable.

I show that the solution to the first-order conditions (13), $R(z;G(z);\alpha d(z)-\beta a(z))$ is an optimum. A sufficient condition is pseudoconcavity. That is, U is increasing in x for $x<R(\cdot)$ and decreasing for $x>R(\cdot)$. Now, take $\tilde{x}<R(\cdot)$ and let $\tilde{z}$ be such that $\tilde{x}=R(\tilde{z})$. Then, $\tilde{z}<z$. Conditional on $dU/dx=0$ and keeping x_{-i} fixed, one has

$$\frac{\partial^2U(x,z-x,S)}{\partial x\partial z}=U_{xy}-U_{yy}+U_{yS}(\alpha(1-G(z))-\beta G(z)). \tag{43}$$

This is greater than zero for $G(z)=0$ and is still greater than zero at $G(z)=1$ for $\beta<(U_{xy}-U_{yy})/U_{ys}>0$ which holds if (41) holds - that is, if β is sufficiently small. Hence, for some $\tilde{x}=x(\tilde{z})<x(z)$, $dU(\tilde{x},z)/dx\geq dU(\tilde{x},\tilde{z})/dx=0$. Thus, U is increasing in x for $x<R(\cdot)$ and we have pseudoconcavity.

[16] This implies the righthand side of the inequality is also a function of β and also of α (through x and μ_X). But given the assumption (v) that $U_{xy}-U_{yy}>0$, and assuming all of the derivatives of U are bounded for finite positive x, it is still clearly the case that there are α, β small enough that the inequality holds.

[17] To give an indication of how restrictive this is, for the log preferences in Section 2.1, when income is distributed uniformly on [1,2], then the constraint is approximately $\alpha+2.2\beta<1$. So, $\alpha=0.5$ and $\beta=0.2$ or $\alpha=1$ and $\beta<0$ would satisfy the constraint, but not $\alpha=\beta=0.5$. Empirical estimates of similar parameters (Drechsel-Grau and Schmid, 2014; Alvarez-Cuadrado et al., 2016) are lower than this.

Now let us consider monotonicity and uniqueness. Applying the implicit function theorem to (13), one has $x'(z) > 0$ if $U_{xy} - U_{yy} + U_{yS}S_x - U_S(\alpha+\beta)g(z) > 0$. Now, S_x is at it lowest at $\bar{z}$ where $S_x = -\beta$. Thus, if the inequality (41) is satisfied, the equilibrium is monotone. Further, given that $x(z)$ is differentiable, by the fundamental theory of differential equations, the differential equations (14) have a unique solution for the given boundary conditions, so that there is exactly one strictly monotone equilibrium.

Further, the monotone equilibrium is the unique equilibrium when $\beta = -\alpha$. I have shown above that strategies must be increasing, but consider a candidate equilibrium that is not strictly monotone so that a mass of agents choose some $\tilde{x}$. The left derivative of S with respect to x then is $\alpha(1 - F_-(\tilde{x})) - \beta F_-(\tilde{x})$, while the right derivative is $\alpha(1 - F(\tilde{x})) - \beta F(\tilde{x})$ with the difference being $(\alpha + \beta)(F(\tilde{x}) - F_-(\tilde{x}))$, where $F_-(\tilde{x}) = \lim_{x \uparrow \tilde{x}} F(x)$. Thus, the difference is zero when $\beta = -\alpha$ and thus (13) is continuous and differentiable. Hence, it could not be the best response for this mass all to choose $\tilde{x}$. So, any equilibrium must be strictly monotone and, by the previous argument, unique.

Equilibrium status $S(x(z); z)$ has total derivative

$$\frac{dS(x(z);z)}{dz} = x'(z)(\alpha(1 - G(z) - \beta G(z)) - (\alpha + \beta)x(z)g(z) - \alpha d'(z) + \beta a'(z).$$

But given (14), this simplifies to $x'(z)(\alpha(1 - G(z) - \beta G(z))$ which is strictly positive for $z < z^*$ and strictly negative for $z > z^*$.

Finally, let us turn to the comparisons with the privately optimal consumption $\hat{x}(z) = \gamma(z, 0)$. For $\beta < 0$, $S_x = \alpha(1 - G(z)) - \beta G(z) > 0$ for all $[\underline{z}, \bar{z}]$; for $\beta = 0$, $S_x > 0$ everywhere except $\bar{z}$ where it is zero. Inspecting the first order condition (13), we have $x(z) \geq \hat{x}(z)$ with equality only where $S_x = S = 0$ which is only the case for $\beta = 0$ and at $\bar{z}$.

For case (i), inequity aversion, I show that $x(z)$ and $\hat{x}(z)$ cross only once and at a point $\tilde{z} > z^* = G^{-1}(\alpha/(\alpha + \beta))$. For $\beta > 0$, $S_x > 0$ for $z < z^*$ and so by the above argument, $x(z) > \hat{x}(z)$ on $[\underline{z}, z^*)$. Given $x(\bar{z}) = \hat{x}(\bar{z})$ when $\beta = 0$, I show $x(\bar{z}) < \hat{x}(\bar{z})$ for $\beta > 0$. Again applying the implicit function theorem to (13), the derivative $\partial x(\bar{z})/\partial\beta$ evaluated at $x = \hat{x}(\bar{z})$ is negative if $(U_{xs} - U_{ys} + S_x U_{ss})(\mu_X - x(\bar{z})) - U_s < 0$, where $\mu_X = a(\bar{z})$ is population average consumption. Clearly, $\mu_X - x(\bar{z}) < 0$. One has $S_x \leq 0$ for $z \geq z^*$ and $U_{ss} \leq 0$ by assumption. By condition (vi), $U_{xs} - U_{ys} = 0$ at $x = \hat{x}(\bar{z})$.[18] Thus, $\partial x(\bar{z})/\partial\beta < 0$, so that an increase in β from zero implies that $x(\bar{z})$ is less than $\hat{x}(\bar{z})$. So there must be a crossing point of $x(z)$ and $\hat{x}(z)$ in $(z^*, \bar{z})$. At any point of crossing on $(z^*, \bar{z})$, one has, again by the implicit function theorem, $x'(z) = (U_{xy} - U_{yy} + U_{ys}S_x - (\alpha + \beta)g(z)U_S)/(b + c) < (U_{xy} - U_{yy})/b = \hat{x}'(z)$, where $b = -U_{xx} - U_{yy} + 2U_{xy} > 0$ and $c = -2S_x(U_{xS} - U_{yS}) - (S_x)^2 U_{SS}$ which is ambiguous in sign because, for $z > z^*$, S_x is negative. If $c > 0$, the inequality follow directly. However, if $c < 0$, the inequality $x'(z) < \hat{x}'(z)$ holds if $b((\alpha + \beta)g(z)U_S - S_x U_{ys}) > (U_{xy} - U_{yy})(-c) > 0$. One can verify that this holds. Thus the crossing is unique. □

[18] For the log preferences of Section 2.1, in contrast $U_{xs} - U_{ys} < 0$. However, one can verify directly from the solution (16) that $x(\bar{z}) < \hat{x}(\bar{z})$ when $\beta > 0$.

Proof of Lemma 2: One needs to choose $x(z)$ to maximize welfare as given in (24). I use the maximum principle approach to maximization, with x as the control variable and d and a as the state variables. The equations of motion are those given in (14). This leads to the Hamiltonian

$$H = g(z)U(x, z-x, x(\alpha(1-G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z)) - \lambda x g(z) + \xi x g(z),$$

where λ and ξ are the costate variables for d and a respectively. Thus the first order condition here is

$$\frac{\partial H}{\partial x} = g(z)(U_x - U_y + U_S S_x) - \lambda(z)g(z) + \xi(z)g(z) = 0. \tag{44}$$

One has further

$$\lambda'(z) = -\frac{\partial H}{\partial d} = \alpha U_S(z)g(z);\ \xi'(z) = -\frac{\partial H}{\partial a} = -\beta U_S(z)g(z).$$

The boundary condition is $\lambda(\underline{z}) = 0$, because the lowest income agent has no negative externality on others through her influence on d. Thus, one has

$$\lambda(z) = \alpha \int_{\underline{z}}^{z} U_S(t)g(t)\ dt = \alpha k(z).$$

Applying a similar method to solve for $\xi(z)$, then substituting into (44), I obtain the first order condition (25) and (26) given in the text. □

Proof of Proposition 2: First, given $\beta \leq 0$, the first order condition for the socially optimal consumption (25) has either one (when $\beta = 0$) or two additional negative terms compared to the Nash equilibrium first order conditions (13). Further, reductions in expenditure by others will reduce $p \equiv \alpha d(z) - \beta a(z)$ and thus the other terms in the first order conditions, $U_x - U_y + U_S S_x$, will also be lower. Thus, $x^*(z) < x(z)$ everywhere.

Second, when $\beta = 0$, at the highest income $x(\bar{z}) = \hat{x}(z)$. Further, one has $S^*(\bar{z}) = S(\bar{z}) = 0$, so that the richest agent has an equal status payoff in the utilitarian optimum as in equilibrium. But by the previous result utilitarian consumption $x^*(\bar{z})$ is lower than equilibrium consumption $x(\bar{z})$ and thus lower than her privately optimal amount $\hat{x}(\bar{z})$. So, one has $U^*(\bar{z}) < U(\bar{z})$. By continuity, this must hold for $\beta < 0$ close to zero. □

Proof of Proposition 3: Start with $\beta = 0$. From Proposition 2, the optimal consumption $x^*(z)$ is everywhere lower than equilibrium consumption $x(z)$. Evaluating (25) at $\bar{z}$, one has the extra negative term $-\alpha k(\bar{z})$, so clearly $x^*(\bar{z}) < x(\bar{z})$. Further, one can show that $x^*(z)$ and $x(z)$ cross at most once. One has by the implicit function theorem, $x'(z) = (U_{xy} - U_{yy} + U_{ys}S_x - (\alpha+\beta)g(z)U_S)/(b+c) > (U_{xy} - U_{yy} + U_{ys}S_x - 2(\alpha+\beta)g(z)U_S)/(b+c) = x^{*\prime}(z)$, where b and c are as defined in the proof of Proposition 1. Thus any point of crossing must be $x^*(z)$ crossing $x(z)$ from above. Thus, either $x^*(z) < x(z)$ for all $z \in [\underline{z}, \bar{z}]$, or $x^*(\underline{z}) > x(\underline{z})$ but $x^*(\bar{z}) < x(\bar{z})$.

Turning to utility, there are two possibilities. Either (more likely) $a^*(\bar{z}) = \mu_X^* \leq \mu_X = a(\bar{z})$, average optimal consumption is lower than average equilibrium consumption

to reduce the negative externality, or possibly $\mu_X^* > \mu_X$ if guilt dominates. In the first case, let $\tilde{x}(\bar{z})$ maximize the richest agent's utility $U(\bar{z}; x, \mu_X^*)$, that is taking the socially optimal average consumption μ_X^* as given. Then, given $\beta > 0$, the agent's utility is increasing in $a(\bar{z}) = \mu_X$, we have $U(\bar{z}) \geq U(\bar{z}; \tilde{x}(\bar{z}), \mu_X^*)$. But also clearly, we have $U(\bar{z}; \tilde{x}(\bar{z}), \mu_X^*) \geq U^*(\bar{z})$ and the result follows. For the other case, where $\mu_X^* > \mu_X$, one can apply a similar argument to show that, because the utility of the individual with income $\underline{z}$ is decreasing in μ_X, she is worse off because $U(\underline{z}) \geq U(\bar{z}; \tilde{x}(\underline{z}), \mu_X^*) \geq U^*(\underline{z})$. □

Proof of Proposition 4: Note that if $\tilde{x}(z) = x(z) - \epsilon$ for all $z \in [\underline{z}, \bar{z}]$, then from the definitions (1) and (3), $dD/d\epsilon = dA/d\epsilon = 0$ and $\tilde{S} = S(x - \epsilon, z) = S(x, z)$. Thus, one has

$$\frac{d\tilde{U}}{d\epsilon} = U_y(\cdot) - U_x(\cdot).$$

For case (a), where $x(z) < \hat{x}(z)$, because there $S_x < 0$ (see Proposition 1), given the equilibrium condition (13), in equilibrium $U_x - U_y > 0$. Thus, for ϵ small, $U_x - U_y > 0$ will also hold and the result follows. For case (b), $S_x > 0$ everywhere, and $U_x - U_y < 0$ for all z in equilibrium. Thus, the policy increases utility. □

Proof of Proposition 5: Note that from (29), $x(1) = Z(1)/2 - \beta\mu_X/(2(1-\beta))$, because $a(1) = \mu_X$, average expenditure on consumption. Further, given $Z_B'(r) > Z_A'(r)$ on $(\hat{r}, 1)$, the maximum income difference is at $r = 1$, so that $Z_B(1) - Z_A(1) > \mu_{XB} - \mu_{XA}$. Thus, one has $x_B(1) > x_A(1)$. Define $p(r) = \alpha d(r) - \beta a(r)$ so that $p'(r) = -(\alpha + \beta)x(r)$ and $p(1) = -\beta\mu_X > 0$. Given $\beta \leq 0$, one has $p(r) > 0$. We have $\mu_B > \mu_A$, average income is higher in B, and this implies through (29), that $\mu_{XB} \geq \mu_{XA}$, average conspicuous consumption is higher. Specifically, integrating (29), one has $\mu_X = \mu/2 + q$ where

$$q = \int_0^1 \frac{\alpha d(r) - \beta a(r)}{2(1 + \alpha(1-r) - \beta r)}\, dr.$$

Thus $\mu_{XA} - \mu_{XB} > 0$ only if $q_A - q_B > \mu_B - \mu_A$. However, note that $q_A - q_B < \alpha\mu_{XA}/(1+\alpha) + \beta\mu_{XB}/(1-\beta) < \mu_{XA} - \mu_{XB} < \mu_B - \mu_A$, given that $\alpha > 0 \geq \beta \geq -\alpha$. Thus, $p_B(1) = \beta a_B(1) = \beta\mu_{XB} \geq p_A(1) = \beta\mu_{XA}$. Then, $x_A(1) < x_B(1)$ implies $p_A(r) < p_B(r)$ for some interval $(\tilde{r}, 1)$ where $\tilde{r}$ is the largest $r \in (0, 1)$ such that $x_A = x_B$ - at any other potential crossing point of p_A and p_B in $(\tilde{r}, 1)$ we have $p_B' = -(\alpha+\beta)x_B < -(\alpha + \beta)x_A = p_A'$. Thus, the greatest crossing point of p_A and p_B in $[0, 1)$ is to the left of the greatest crossing point of x_A and x_B. This in turn implies that $x_A(r) < x_B(r)$ everywhere on $[0, 1]$ from (29) because $p_B(r) > p_A(r)$ and $Z_A(r) \leq Z_B(r)$.

Turning to utility, one has, after some manipulation,

$$U(r) = \ln[1 + \alpha(1-r) - \beta r] + 2\ln[y(r)],$$

where non-conspicuous consumption $y(r) = Z(r) - x(r) = Z(r)/2 - (\alpha d(r) - \beta a(r))/(2(1+\alpha(1-r) - \beta r))$. Thus, U_A and U_B cross only when y_A crosses y_B. Further, $y'(r) = Z'(r)/2 + (\alpha+\beta)y(r)/(2(1+\alpha(1-r) - \beta r))$ so that at a point of crossing of y_A and y_B, the relative slopes are determined by comparing Z_A' and Z_B'. We have $Z_A'(r) < Z_B'(r)$ on $(\hat{r}, 1)$ so that there is at most one point of crossing of y_A and y_B and thus at most

one crossing of U_A and U_B. Clearly, $U_A(1) < U_B(1)$ because again $Z_B(1) - Z_A(1) > \mu_{XB} - \mu_{XA}$ so that $y_B(1) > y_A(1)$. But, one can see that, because income at $\hat{r}$ is unchanged but $p_A(\hat{r}) < p_B(\hat{r})$ as shown above, then we have $U_A(\hat{r}) > U_B(\hat{r})$. So the only crossing of U_A and U_B is on $(\hat{r}, 1)$ and not on $(0, \hat{r})$. □

Proof of Proposition 7: It will be useful to differentiate the equilibrium consumption function (16) with respect to z, using the derivatives given in (14), to obtain,

$$x'(z) = \frac{1}{2} - \frac{(\alpha+\beta)g(z)(z-x)}{2(1+\alpha(1-G(z)) - \beta G(z))} = \frac{1}{2} + \frac{(z-x)}{2}\phi(z), \tag{45}$$

where $\phi(z) = -(\alpha+\beta)g(z)/(1+\alpha-(\alpha+\beta)G(z)) < 0$. Define

$$Q(z) = \frac{G_A(z) - \frac{1+\alpha}{\alpha+\beta}}{G_B(z) - \frac{1+\alpha}{\alpha+\beta}} \tag{46}$$

Note that $(1+\alpha)/(\alpha+\beta)$ is greater than one and thus $Q(z) > 0$ for all $[\underline{z}, \bar{z}]$. Given $L(z) = g_A(z)/g_B(z)$, one has

$$\frac{\phi_A(z)}{\alpha+\beta} = \frac{g_A(z)}{G_A(z) - \frac{1+\alpha}{\alpha+\beta}} < \frac{g_B(z)}{G_B(z) - \frac{1+\alpha}{\alpha+\beta}} = \frac{\phi_B(z)}{\alpha+\beta} \Leftrightarrow L(z) > Q(z). \tag{47}$$

The following result is effectively the reverse of Lemma A2 in Hopkins and Kornienko (2004).

Lemma 3. *If $G_A(z) \succ_{ULR} G_B(z)$ then for all $\alpha \geq 0$, $Q(z)$ has two extremes, a maximum at $\hat{z}_-$ and a minimum at $\hat{z}_+$, such that $\underline{z} \leq \hat{z}_- < \hat{z} < \hat{z}_+ < \bar{z}$. Further $L(z)$ and $Q(z)$ cross only at $\hat{z}_-$ and $\hat{z}_+$. Thus, $\phi_A(z) > \phi_B(z)$ on both $[\underline{z}, \hat{z}_-)$ and $(\hat{z}_+, \bar{z}]$ but $\phi_A(z) < \phi_B(z)$ on $(\hat{z}_-, \hat{z}_+)$.*

Proof. Note that $Q(\underline{z}) = Q(\hat{z}) = Q(\bar{z}) = 1$. Therefore, there is at least one extreme point for $Q(z)$ on each of the two intervals $(\underline{z}, \hat{z})$ and $(\hat{z}, \bar{z})$. Note that $dQ(z)/dz = 0$ if and only if $Q(z) = L(z)$. That is, $Q(z)$ and $L(z)$ cross at the turning points of $Q(z)$. Now since $L(z)$ is increasing on $(\underline{z}, \hat{z})$, at any crossing point $L(z)$ must cross $Q(z)$ from below and so there can only be one crossing point on $(\underline{z}, \hat{z})$. Equally there can be only one extreme for $Q(z)$ on $(\hat{z}, \underline{z})$. Lastly, $Q(z)$ is increasing iff $L(z) < Q(z)$ and we can see that the maximum is at $\hat{z}_-$ and the minimum at $\hat{z}_+$. The final result then follows from (47). □

(a) Because, by definition $d_A(\bar{z}) = d_B(\bar{z}) = 0$, we have from (16) that $x_A(\bar{z}) = x_B(\bar{z})$. Second, because the ULR order implies that $g_A(\bar{z}) < g_B(\bar{z})$ (and $G_A(\bar{z}) = G_B(\bar{z}) = 1$), by (45) we have $x'_A(\bar{z}) > x'_B(\bar{z})$. Thus, $x_A(z)$ crosses $x_B(z)$ from below at $\bar{z}$ so that $x_A(z) < x_B(z)$ on $(\bar{z} - \varepsilon, \bar{z})$ for some $\varepsilon > 0$. Third, there can be no crossing point in $(\hat{z}_+, \bar{z})$. Suppose there was such a point then simultaneously one would have $x_A = x_B$ and $x'_A < x'_B$. But $\phi_A(z) > \phi_B(z) \in (\hat{z}_+, \bar{z})$ by Lemma 3, so that $x'_A > x'_B$ wherever $x_A = x_B$ on this interval. Fourth, there can be a crossing point on $(\hat{z}_-, \hat{z}_+)$ because there $\phi_A(z) < \phi_B(z)$. If there is a crossing on $(\hat{z}_-, \hat{z}_+)$, then there can be a crossing on

$(\underline{z}, \hat{z}_-)$, with $x'_A > x'_B$ because $\phi_A(z) > \phi_B(z)$ on this interval.
(b) Again because $d_A(\bar{z}) = d_B(\bar{z}) = 0$, and $x_A(\bar{z}) = x_B(\bar{z})$, it follows that $U_A(\bar{z}) = U_B(\bar{z})$. We have by the envelope theorem

$$U'(z) = \frac{1}{z - x};\ U''(z) = \frac{x'(z) - 1}{(z - x)^2}. \tag{48}$$

So, $U'_A(\bar{z}) = U'_B(\bar{z})$, but as shown above we have $x'_A(\bar{z}) > x'_B(\bar{z})$ so that $U''_A(\bar{z}) > U''_B(\bar{z})$ so that $U_A(z) > U_B(z)$ for z immediately lower than $\bar{z}$. First, take the case where $x_A(z) < x_B(z)$ everywhere on $(\underline{z}, \bar{z})$. Then at any potential crossing point of U_A and U_B we would have $U'_A(z) < U'_B(z)$ so there could only be one such crossing point and U_B would cross U_A from below. But such a crossing point would contradict that $U_A(z) > U_B(z)$ in the neighborhood of $\bar{z}$. Second, suppose x_A crosses x_B at some point $\tilde{z} < \hat{z}$. By the above argument we have $U_A(z) > U_B(z)$ on $(\tilde{z}, \bar{z})$. In fact, at $\tilde{z}$ the function $U_A(z) - U_B(z)$ has its maximum. Since $U_A(\tilde{z}) - U_B(\tilde{z}) > 0$, and because $x_A(\tilde{z}) = x_B(\tilde{z})$ so that $\alpha d_A(z)/(1 + \alpha(1 - G_A(z)) = \alpha d_B(z)/(1 + \alpha(1 - G_B(z))$, it must be from (17) that $G_A(\tilde{z}) < G_B(\tilde{z})$ so that it follows that $\tilde{z} < \hat{z}$. Further, if there is further crossing point of x_A and x_B in $(\underline{z}, \hat{z}_-)$ then this is a minimum for $U_A - U_B$ and it also holds that $U_A - U_B > 0$ there, so that $U_A(z) > U_B(z)$ for all of $[\underline{z}, \bar{z})$. □

Proof of Proposition 8: (a) In this case $x_A(\underline{z}) = x_B(\underline{z}) = \underline{z}/2$. We have $g_B(z) > g_A(z)$ on $[\underline{z}, \hat{z}_-)$ so that, from (45) and given $\beta < 0$, it follows that $x'_B(\underline{z}) > x'_A(\underline{z})$, implying $x_B(z) > x_A(z)$ in the neighborhood of $\underline{z}$, and second, one would have $x'_B > x'_A$ at any potential crossing point in the interval $[\underline{z}, \hat{z}_-)$. So there can be no further crossing until $z > \hat{z}_-$.
(b) Since $x_A(\underline{z}) = x_B(\underline{z}) = \underline{z}/2$, one has $U_A(\underline{z}) = U_B(\underline{z})$ and $U'_A(\underline{z}) = U'_B(\underline{z})$ from (48). But because $x'_B(\underline{z}) > x'_A(\underline{z})$, one has $U''_A(\underline{z}) < U''_B(\underline{z})$ so that $U_B > U_A$ in the neighborhood of $\underline{z}$. Further, since $x_B > x_A$ on $(\underline{z}, \hat{z}_-)$, there can be no further crossing. □

Proof of Proposition 11: First, an equilibrium strategy $x(z)$ is necessarily weakly increasing as shown in the proof of Proposition 1. Second, $x(z)$ is continuous. Given the previous result, only upward jumps are potentially possible. The first order condition is continuous and differentiable in x where $F(x)$ is continuous and differentiable.[19] Thus, it implicitly defines a continuous $x(z)$. If there were an upward jump in x, say from x_1 to x_2, then $F(x)$ would be constant on $[x_1, x_2]$ and thus continuous and differentiable. Thus, $x(z)$ would be continuous, a contradiction. Finally, $x(z)$ will have the same qualitative properties as a strictly monotone equilibrium wherever $F(x)$ is continuous and thus the first order conditions (8) and (13) are identical. But if $F(x)$ is discontinuous at some point $\tilde{x}$, from (8), the left derivative S_x is positive for sure if $F_-(x) < \alpha/(\alpha+\beta)$. Then $x(z) > \hat{x}(z)$ for $x \leq \tilde{x}$ by an adaptation of the argument in Proposition 1. In the inequity averse case where $\beta > 0$, when $F(x) = 1$, then $d(x; x_{-i}) = 0$ but $a(x; x_{-i}) > 0$ and so $x < \hat{x}$. □

[19] At points where $F(x)$ is discontinuous, $F(x)$ jumps upwards because a mass of agents on an interval, say $[\tilde{z}_-, \tilde{z}_+]$, choose the same $\tilde{x}$. That is, $F(x)$ is discontinuous but $x(z)$ is constant and thus continuous on $[\tilde{z}_-, \tilde{z}_+]$.

Proof of Proposition 12: Given the new expression for S_x given in (32), one can see that, if $\alpha > 0 \geq \beta$, then clearly $S_x > 0$ and S_x is increasing in θ. The result then follows directly by applying the methods in the proof of Proposition 1. □

Proof of Proposition 13: Existence can be derived from the proof of Proposition 1. The comparison with the privately optimal case ($\alpha = \beta = 0$) follows simply from inspection of (38), (39) and (40). The comparison with the socially optimal schedules follows from Proposition 2. □

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